Hydraulic Conductivity for Porous Medium with Wavy 3-D Pores – 16244

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ABSTRACT

The flow of a viscous fluid is considered to calculate the permeability of a porous medium with wavy pores. Under the assumption of periodicity a unit cell which contains solid phase of wavy shape is chosen on the microscale and the flow field is calculated by solving a certain boundary-value problem (BVP), Stokes problem. The BVP is obtained by applying the theory of homogenization to slow viscous flow through porous structure on the microscale. It is shown that the permeability is dominantly determined by the flow through straightly connected portion of the pore space.

INTRODUCTION

When there is an externally imposed pressure gradient over a porous medium on the macroscale, the fluid in the pores is driven to flow through the medium as a result of balance between the pressure gradient and the viscous drag exerted by the solid boundary on the fluid. It is important to estimate the permeability of the medium. For effective management of the underground repository it is essentially important to know the flow characteristics which may lead to the determination (or reliable estimation) of the medium permeability. Knowledge of flow characteristics will also enable one to readily investigate the transport of contaminant released in the medium.

In this study, a medium which is composed of wavy solid and fluid phases is chosen and the flow field in the pore space is calculated from which the permeability is determined. The calculations are carried out by using FLUENT packed in ANSYS. The theoretical framework is based on the homogenization theory which systematically combines the processes on the microscale and deduces the governing equations and the effective coefficients on the macroscale [1]. Under two basic assumptions, (i) the periodicity of the medium structure on the microscale with periodic length ℓ and (ii) the periodicity of all variables and material properties over the same length. It is noted that the periodicity assumption is not very restrictive because the distributions and arrangements over the periodic length are quite arbitrary.

When there is an externally imposed pressure gradient in the x-direction, it is shown that the permeability is dominantly influenced by the flow field in the straight wavy pores that connect through the medium.

It is emphasized that the approach in the present study does not assume any ad hoc or phenomenological assumptions, except the periodicity assumption which is in a sense not restrictive, and starts from the basic governing laws on the microscale and deduces the effective relations on the macroscale systematically by using the multiple-scale perturbation procedure. The permeability is calculated, not estimated, from the solution to the Stokes problem, a boundary-value problem defined in in the pore space of a microcell.

THE GOVERNING RELATIONS ON THE MICROSCALE

The porous medium is composed of the solid phase (Ωs) and the fluid phase (Ω_f) which is saturated by a liquid. The microscale cell domain is represented by $\Omega = \Omega s + \Omega_f$. Each phase is assumed to be connected throughout the porous medium. Fluid flow is induced by a pressure gradient imposed over the medium on the macroscale. Hence, on the microscale, the leading order pressure is linearly varying with microscale correction which is determined by solving a microcell boundary-value problem.

The basic governing relations and the boundary conditions that must be satisfied in the fluid domain Ω_f are desribed without showing the explicit forms..

The governing equations for the fluid on the microscale in the fluid phase (Ω_f) are the conservation laws of mass and momentum[1].

On the boundary Γ between the solid and fluid, the liquid velocity vanishes

The governing equations and the boundary conditions are normalized by using the representative scales. It is assumed the inertial effects are small so that the Reynolds number is small.

MULTIPLE SCALE ANALYSIS

The distinguishing features of the multiple scale perturbation analysis are briefly summarized. In view of the scale disparity of the porous medium, two distinct length scales are introduced: the microscale - the fast scale which is equivalent to the representative elementary volume in the traditional treatment of the process and the macroscale - the scale over which the processes of interest take place from the viewpoint of reservoir engineering and management.

The variables are expanded as perturbation series in the following small parameter

$$\frac{\ell}{\ell'} = \epsilon \ll 1 \qquad \text{(Eq. 1)}$$

in which ℓ is the microscale length and ℓ' the macroscale length. Expanding the governing equations and boundary conditions, the microscale boundary-value problems are investigated separately according to the respective order of ϵ and, through volume-averaging over the micro-cell, the effective macroscale governign equations are derived.

In the process of the multiple scale analysis, a canonical micro-cell boundary-value problem is defined whose solution is used in the calculation of the effective macroscale coefficients by averaging over the micro-cell volume.

THE BOUNDARY-VALUE PROBLEM IN THE UNIT CELL

For a porous medium with periodic arrays of unit cells a boundary-value problem is defined during the process of applying the multiple-scale expansions to the basic governing relations on the microscale. Specifically the fluid pressure at the leading order is shown to be independent of the microscale. The fluid velocity in the pore and the correction for pressure are then represented in terms of the macroscale pressure gradient as[2]

$$\begin{aligned} \mathbf{v}^{(0)} &= -\mathbf{K} \cdot \nabla' p^{(0)} \\ p^{(1)} &= -\mathbf{S} \cdot \nabla' p^{(0)} \end{aligned}$$

(Eq. 2a, b)

in which dimensionless variables are used and the primed gradient operator is with respect to the macroscale.

For fluid flow the following Stokes problem in dimensionless variables is defined:

$$\begin{aligned} \nabla^2\mathbf{K} - \nabla\mathbf{S} + \mathbf{I} &= 0 \quad \text{in} \quad \Omega_f \\ \nabla \cdot \mathbf{K} &= 0 \quad \text{in} \quad \Omega_f \\ \mathbf{K} &= 0 \quad \text{on} \quad \Gamma \\ \langle \mathbf{S} \rangle &= 0 \\ \mathbf{K} \text{ and } \mathbf{S} \text{ are} \quad & \Omega- \quad \text{periodic.} \end{aligned}$$

(Eq. 3a-e)

In the above, **K**=K_{ij} and **S**=S_j are the fluid velocity in the i-th direction and the fluid pressure variation in the micro-cell due to externally imposed pressure gradient in the j-th direction. The unprimed gradient operator is with respect to the microscale coordinates. The pair of angle brackets in (Eq. 3d) is the volume average over Ω as defined below in (Eq. 4).

Equations (Eq. 3a) and (Eq. 3b) are the momentum conservation of the fluid driven by a unit force with no convective inertia and the continuity equation, respectively. The no-slip condition on the fluid-solid interface is given in (Eq. 3c). Equation (Eq. 3d) is imposed to ensure the uniqueness of the pressure. Lastly (Eq. 3e) is imposed to satisfy the periodicity condition.

The macroscale permeability tensor of rank two is then given by the micro-cell volume average of **K** as

$$\langle \mathbf{K} \rangle = \frac{1}{\Omega} \int_{\Omega} \mathbf{K} d\Omega$$

(Eq. 4)

and the Darcy's law is given as

$$\langle \mathbf{v}^{(0)} \rangle = -\langle \mathbf{K} \rangle \cdot \nabla' p^{(0)}$$

(Eq. 5)

where the left-hand side is the seepage velocity and the primed gradient is the derivative of the fluid pressure over the macroscale. This serves as the momentum equation on the macroscale.

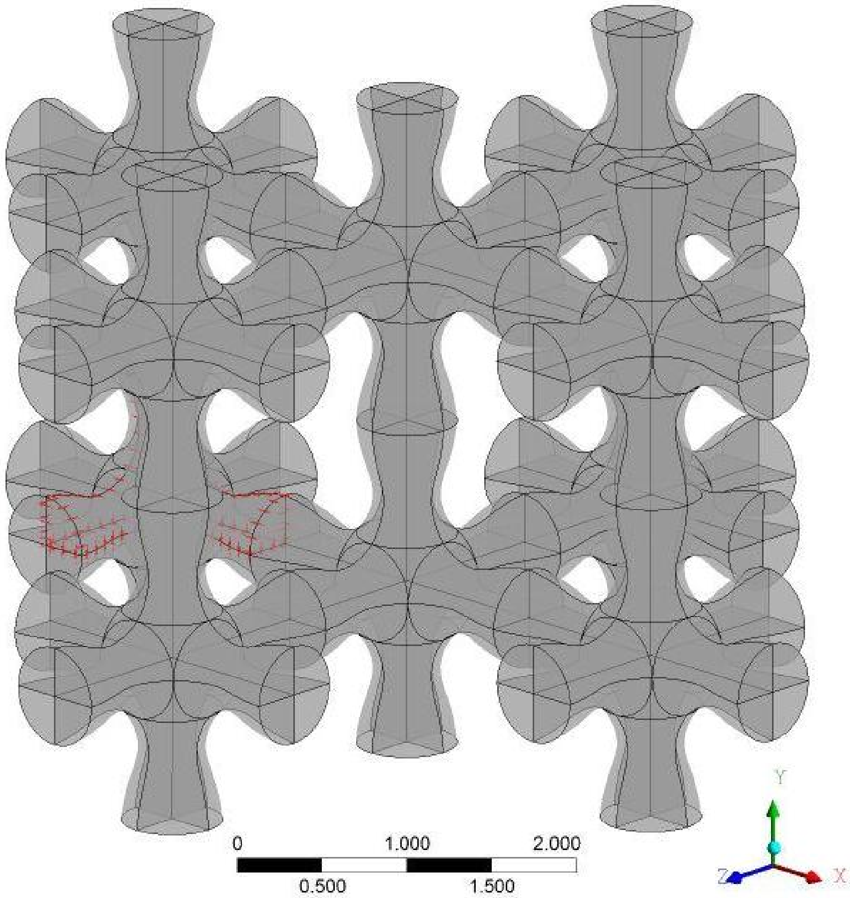
THE DISCRETIZATION

The geometry of the unit cell on the microscale is as shown in Fig. 1 in which wavy solid and fluid phases are shown.

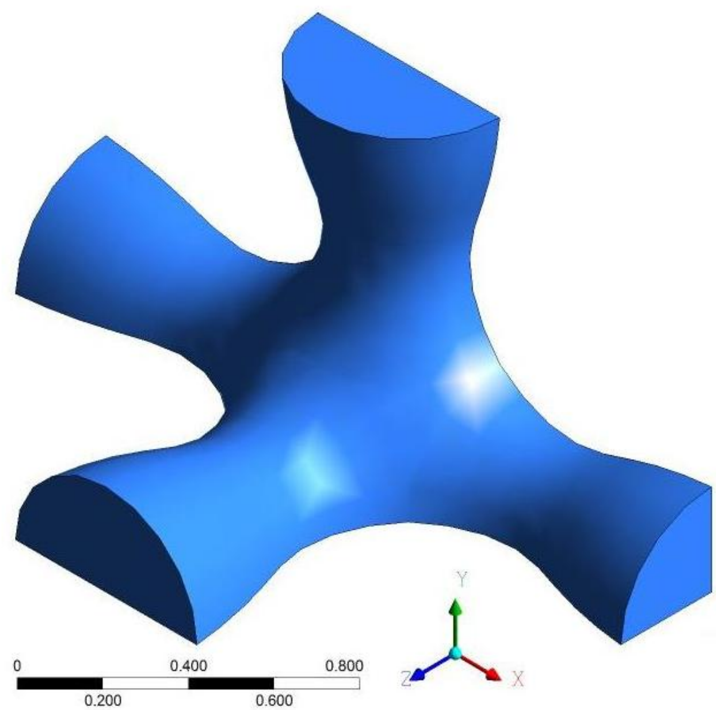
Except for simple and elementary geometries the Stokes problem does not allow analytic solution and has to be solved numerically. For this purpose the commercial software FLUENT for flow analysis (packed together with other purpose ones in ANSYS) has been used.

In solving the Stokes problem, three progressively finer finite element meshes were used. They are labelled as Mesh 0.1, 0.05, and 0.03 respectively. The numbers in the mesh label denotes the representative element size in the discretization along both the horizontal and vertical sides of the unit cell. As the number decreases, the number of finite elements increases sharply in proportion to the inverse of the cubic of the label number.

The three meshes are shown in Fig.2.

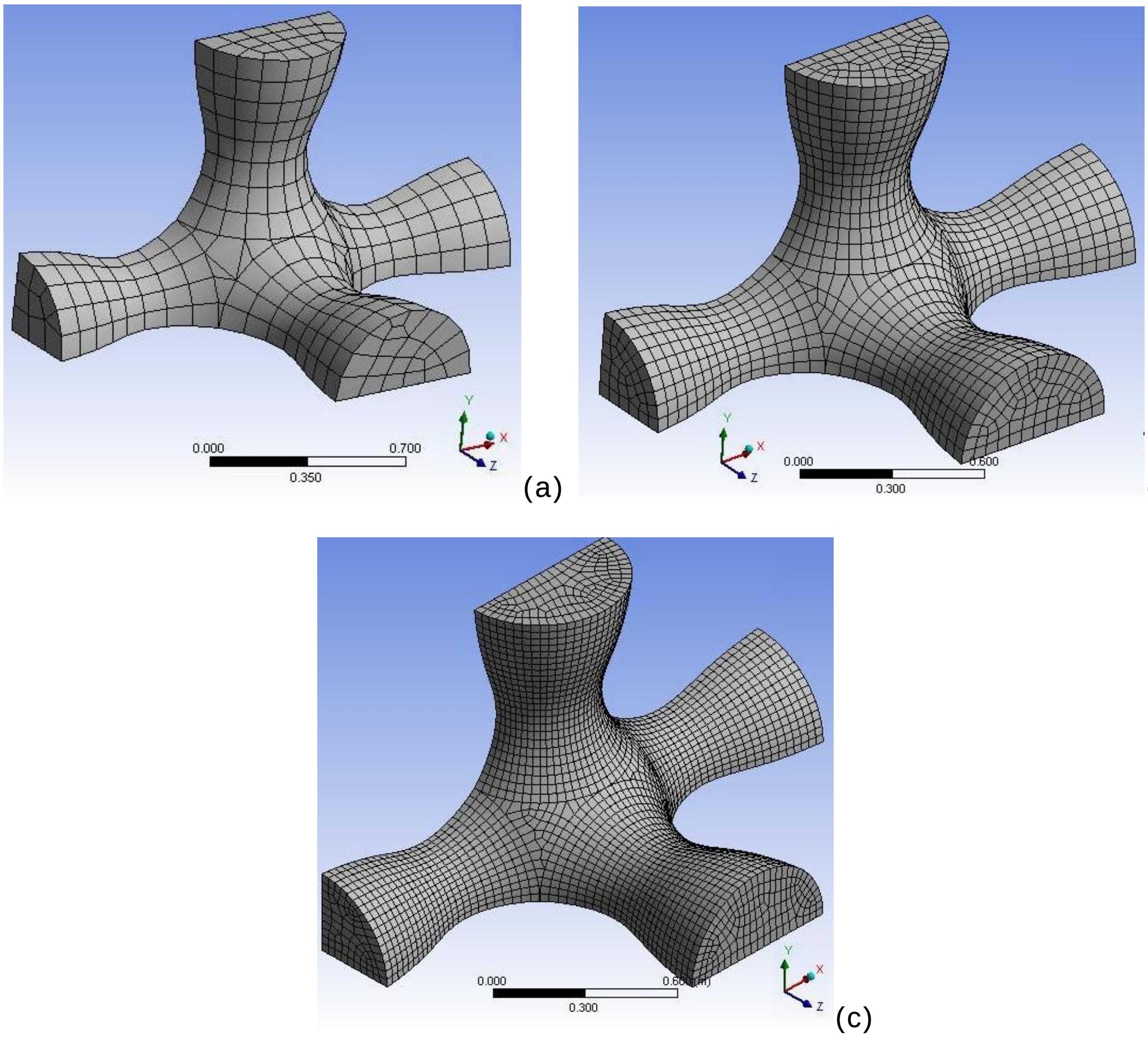


(a)



(b)

Fig.1. Geometry of the porous medium: (a) Array of microcells and (b) One-eighth of a microcell.



(a) (b) (c)

Fig.2. Three meshes used in solving the Stokes problem: (a) Mesh 0.1, (b) Mesh 0.05, and (c) Mesh 0.03.

THE FLUID VELOCITY AND PRESSURE DISTRIBUTIONS

The velocity distribution in the pores determined from the finest mesh(Mesh 0.03 shown in Fig. 2(c)) is shown in Fig. 3 which has been adjusted automatically by CFX for clear and convenient display of the velocity arrows.

Due to the external pressure gradient in the x-direction the velocity vestors are aligned predominantly in the x-direction. Notice that, due to the periodicity condition, (Eq. 3e), the velocity arrows on the left boundary are repeated on the corresponding portions of the right boundary. Notice that, due to symmetry in the geometry, the velocity is nearly zero in the upper part of the pore space which is normal to the x-direction. Hence the upper part of the pore is practically isolated from other part from the flow view point. Due to the no-slip condition for the velocity, (Eq. 3c), the fluid velocity vanishes on the surface of the grains.

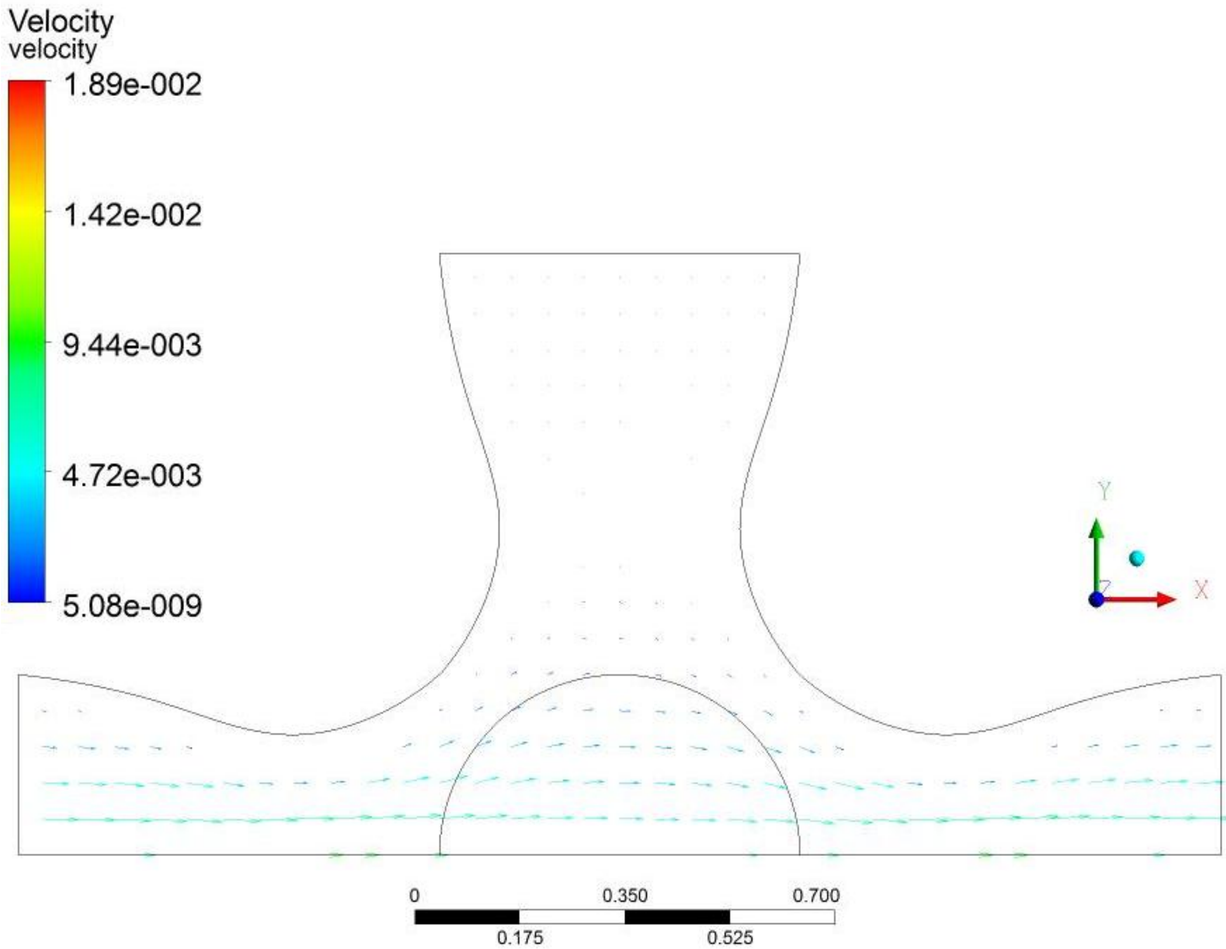


Fig. 3. Velocity distribution in the pores (Mesh 0.03) when the external pressure gradient is in the x-direction.

The pressure distribution in the pore space is shown in Fig. 4. Notice that the pressure contour lines are quite vertical in the central zone especially in the upper wavy pore implying that the velocity has nearly vanished. It is consistent with the observation of the velocity distribution shown in Fig. 3 and discussed above.

On the other hand, in the channel-like wavy pore in the lower region along x-direction the pressure changes along the channel direction with relatively large change where, in fact, the velocity takes the largest values as seen in Fig. 3.

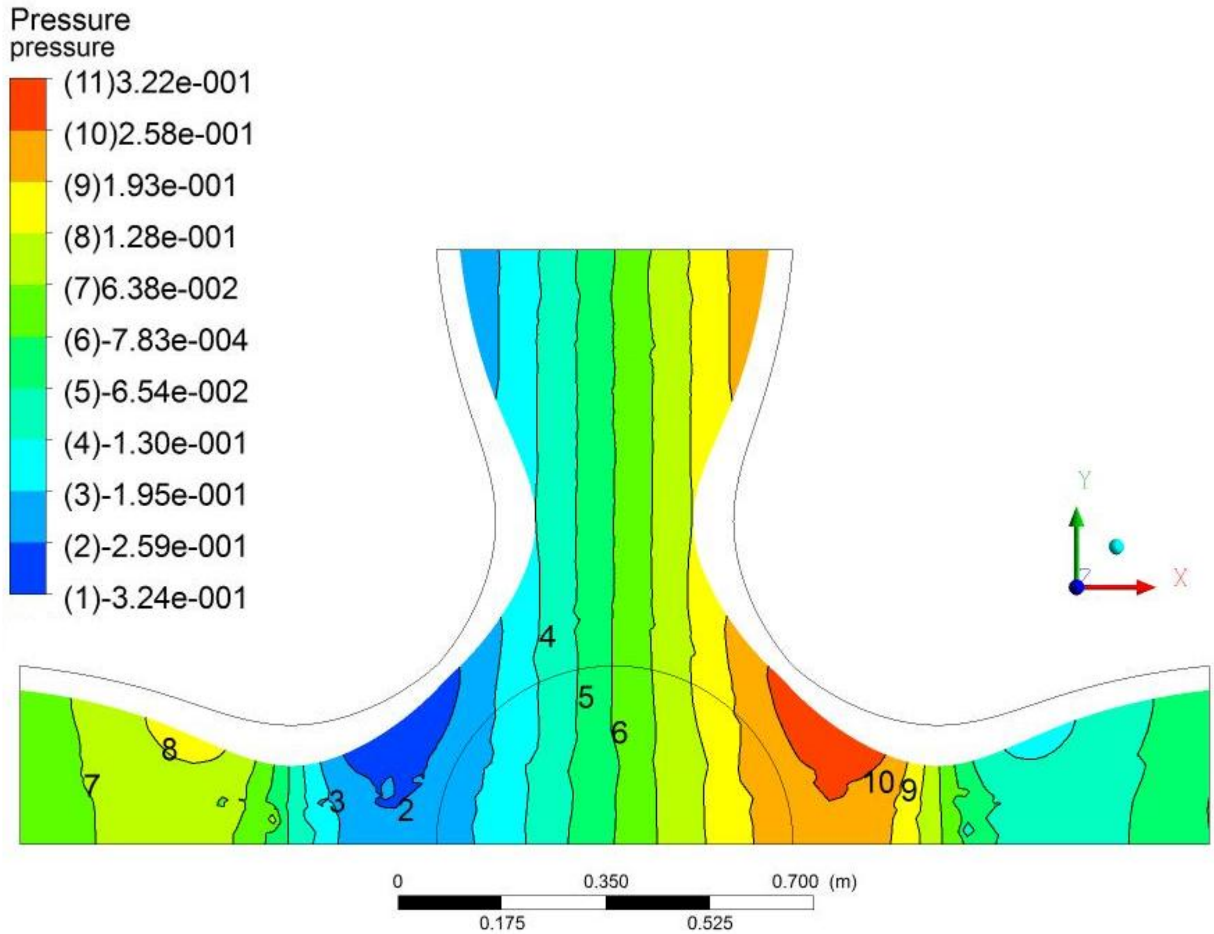


Fig. 4. Pressure distribution in the pores when the external pressure gradient is in the x-direction.

RESULTS AND DISCUSSION

The permeability is calculated from the velocity field by taking the volume average over the micro-cells using, cf. (Eq. 4).

The values calculated from the four different meshes are summarized in Table 1 below.

Table 1 Permeability and porosity values from different meshes

Mesh type	Permeability	Porosity
0.1	2.94837E-03	0.25218
0.05	2.84209E-03	0.254341
0.03	2.78668E-03	0.254698

The permeability <Kxx>, the permeability in the x-direction due to the macroscale pressure gradient in the x-direction, converges quite well. The error is less than 2% and the result is quite accurate enough. The permeability is dominantly contributed by the flow field along the central wavy pore in Fig. 3.

The porosity value converges quickly as the mesh is refined, i.e., as the grain boundaries become smooth with the refinement of discretization.

The microcell geometry used in the present study is composed of wavy solid and fluid regions. By modifying the waviness it can be used to model a medium composed of round grains.

CONCLUSIONS

From the calculations of the permeability in a porous medium with wavy pores and solid phase in a unit cell on the microscale the following conclusions are drawn.

1. The permeability of a medium with wavy pores is dominantly influenced by the straight connecting portions of the pore space.
2. Calculation of permeability for a porous medium with general geometry, but periodic on the microscale, can be carried out efficiently by using finite elements.
4. It is worthwhile extending the computational procedure in this study to medium structures with grains of varying waviness and other distribution patterns.

REFERENCES

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