Flow in and Permeability of Porous Media for Turbulent Flow – 16117

Cheo Kyung Lee *, Sophy Sok *, Sung Paal Yim **

*** Handong Global University**
3 Namsong-ri, Heunghae-eub, Buk-gu, Pohang, Kyungbuk, 791-708
Republic of Korea

**** Korea Atomic Energy Research Institute**
P.O. Box 150, Yusong, Daejon, 305-600
Republic of Korea

ABSTRACT

The turbulent flow of an incompressible fluid in a porous medium with irregularly shaped grains is analyzed by adopting the k-ε model and making use of ANSYS. The effective permeability is dominantly in the longitudinal direction with negligibly small transverse component due to anisotropic arrangement of the grains. It is observed that the turbulent kinetic energy is large in the central region of pores, but the turbulent dissipation appears to be quite uniform.

INTRODUCTION

The subsurface environment is composed of porous media that are sometimes characterized as granular medium. The shapes of the granular particles often appear to be irregular. The void space is occupied by either a liquid or a gas or both.

In many cases, production wells are installed and operated. Although the flow of a fluid in an underground environment is mostly laminar, it becomes quite intense near extraction wells and may easily reach turbulent regime. Hence it is important to investigate turbulent flow patterns in the neighborhood of production or injection wells.

In this study, turbulent flow of an incompressible fluid in porous media is studied to calculate the effective permeability. For this purpose, the k-ε model is adopted and the calculations are performed by using a commercial software ANSYS package.

It is shown that the turbulent kinetic energy is large in the regions away from solid boundaries and decreases sharply near the boundary. The turbulent dissipation is however quite uniform in the pores and is merely the consequence of the existence of turbulence. The average fluid velocity is dominantly in the direction of the externally imposed pressure gradient whereas the pressure distribution shows some varying patterns with relatively large values in the regions confronting the flow.

For the effective permeability, it is dominantly in the longitudinal direction with very small transverse value which is believed to be the reflection of certain degree of anisotropy in the arrangement of the solid grains.

THE GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The Governing Equations

The turbulent flow is analyzed by using k-ε model which states the transport of the kinetic energy (k) and the dissipation (ε) due to fluctuating velocity components[1]. The governing equations are[2]

$$\frac{\partial}{\partial t}(\rho \mathrm{k}) + \frac{\partial}{\partial x_i}(\rho k \mathrm{u}_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\varepsilon - Y_M + S_k$$

and

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \frac{\partial}{\partial x_i}(\rho\varepsilon \mathrm{u}_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{1\varepsilon}\frac{\varepsilon}{k}(G_k + C_{3\varepsilon}G_b) - C_{2\varepsilon}\rho\frac{\varepsilon^2}{k} + S_\varepsilon$$

where

$$\mathrm{k} = \ 1/2 < u'.u' >$$

and

$$\varepsilon \equiv 2v < s_{ij}s_{ij} >$$

The time average has been denoted by a pair of angle brackets.

where u' is the velocity fluctuation about the mean and s_{ij} is the shear strain due to the fluctuating velocity components and v is the kinematic viscosity of the fluid. The turbulent (or eddy) viscosity, μ_t, is computed by combining k and ε as follows:

$$\mu_t = \rho \mathrm{C}_\mu \frac{k^2}{\varepsilon}$$

Also the following are defined:

G_k : the generation of turbulence kinetic energy due to the mean velocity gradients

G_b : the generation of turbulence kinetic energy due to buoyancy

Y_M : the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate

S_k , S_ε : user-defined source terms

σ_k , σ_ε : the turbulent Prandtl numbers for k and ε

$C_{1\varepsilon}$,$C_{2\varepsilon}$, , $C_{3\varepsilon}$: constants.

Specifically the following values are chosen:

$$C_{1\varepsilon} = 1.44,\ C_{2\varepsilon} = 1.92,\ C_{3_\varepsilon} = 1,\ C_\mu = 0.09,\ \sigma_k = 1.0,\ \sigma_\varepsilon = 1.3$$

Also

$$G_b = \beta g_i \frac{\mu_t \partial T}{Pr_t \partial x_i}$$

$$G_k = -\rho \overline{u_i' u_j'} \frac{\partial u_j}{\partial_{x_i}}$$

$$Y_M = 2\rho\varepsilon M_t^2$$

where Pr_t is the turbulent Prandtl number for energy, β is the thermal expansion coefficient, and M_t is the turbulent Mach number.

The Boundary Conditions

There are two types of boundary conditions: the wall condition on the interfaces between the solid and fluid regions inside the microcell and the periodic boundary conditions on the outer boundaries of the microcell Also all the variables and coefficients are assumed to be periodic on the microscale, i.e., they are periodic in the coordinate directions with certain periodic lengths.

The boundary conditions are shown schematically in Fig. 1(a) and (b).

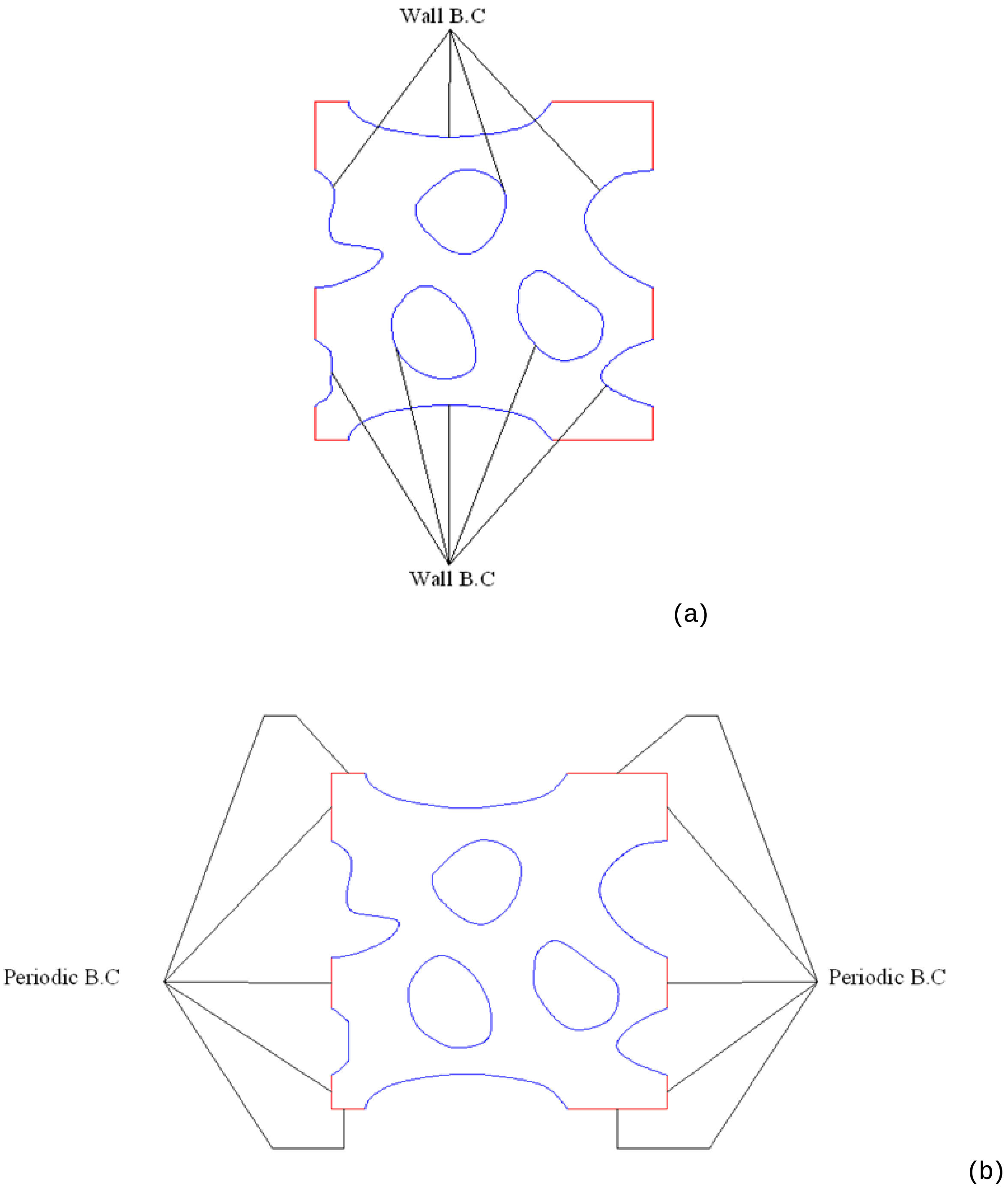


Fig. 1. (a) The wall and (b) the periodic boundary conditions

NUMERICAL COMPUTATION

THE MICROCELL GEOMETRY, DISCRETIZATION, AND NUMERICAL SOLVER

The geometry of the unit cell on the microscale is as shown in Fig. 2 in which six irregularly shaped solid grains are shown. Three particles located across the boundary are reappearing after the microscale length: two irregular shaped grains along the horizontal direction and one elliptical shape in the vertical direction. The solid grains are shown in empty shapes and the pore space is shown with discretized mesh.

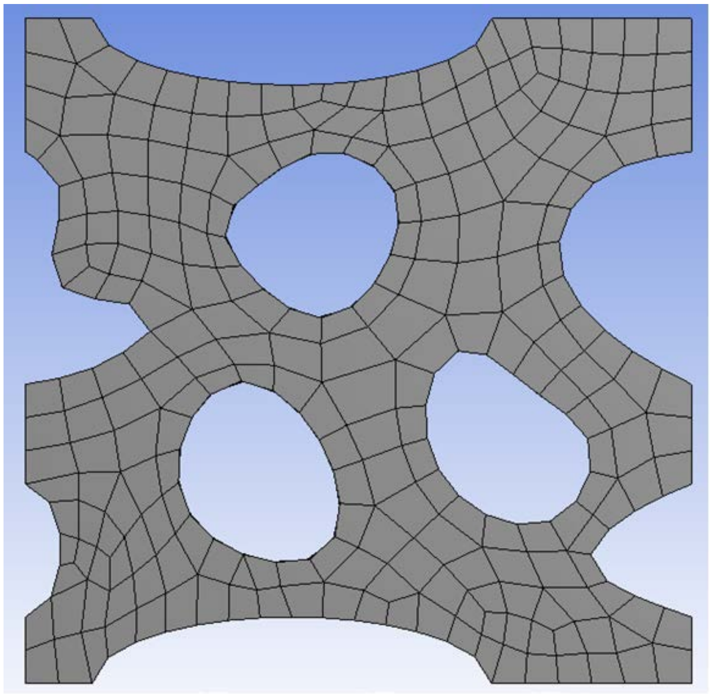


Fig.2. A finite element mesh used in solving the k-ε model in FLUENT.

The commercial software FLUENT (packed in ANSYS) has been used in solving the k-ε model for turbulent flow analysis.

RESULTS AND DISCUSSIONS

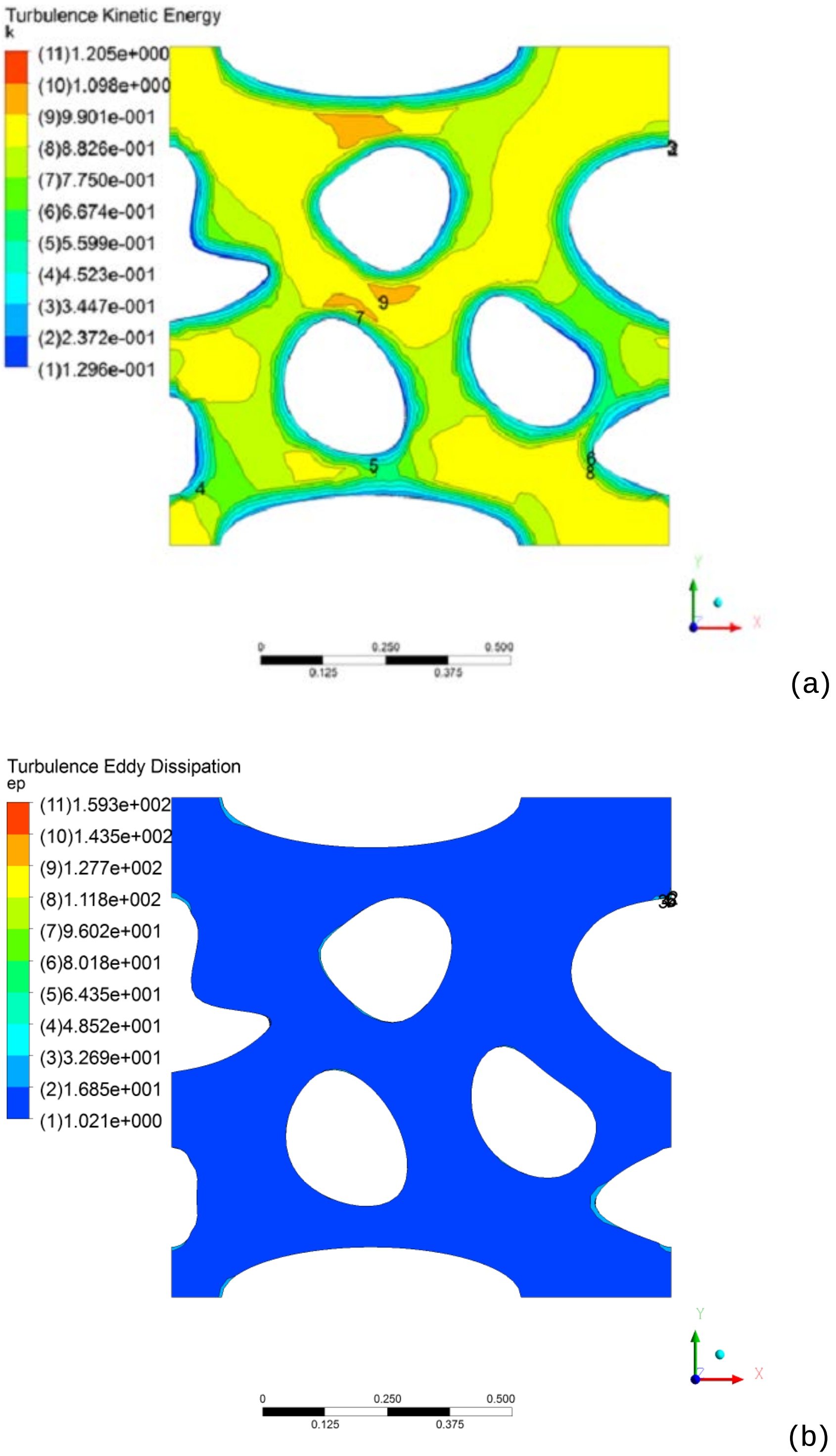
The fluid that passes through the pores is assumed to be water. The results obtained are shown below for the turbulent kinetic energy, dissipation, the velocity and the pressure when a unit pressure gradient is imposed in the *x*-direction.

Turbulence Kinetic Energy and Dissipation

The distributions of the turbulent kinetic energy k and the dissipation ε are shown in Fig. 3 (a) and (b).

The magnitude of k is clearly larger in the middle of the pores away from the grain boundaries and decreases near the interface between the solid grain and the fluid. Also the intensity of k becomes the largest in the narrow channels where the mean velocity (time-averaged) is the largest.

The magnitude of ε is quite uniform in the pore space implying that it is nearly not affected by the shape of the pores nor the existence of bounding solid particles. Hence it is the consequence of just the nature of fluid flow belonging to a turbulent regime.



(a)

(b)

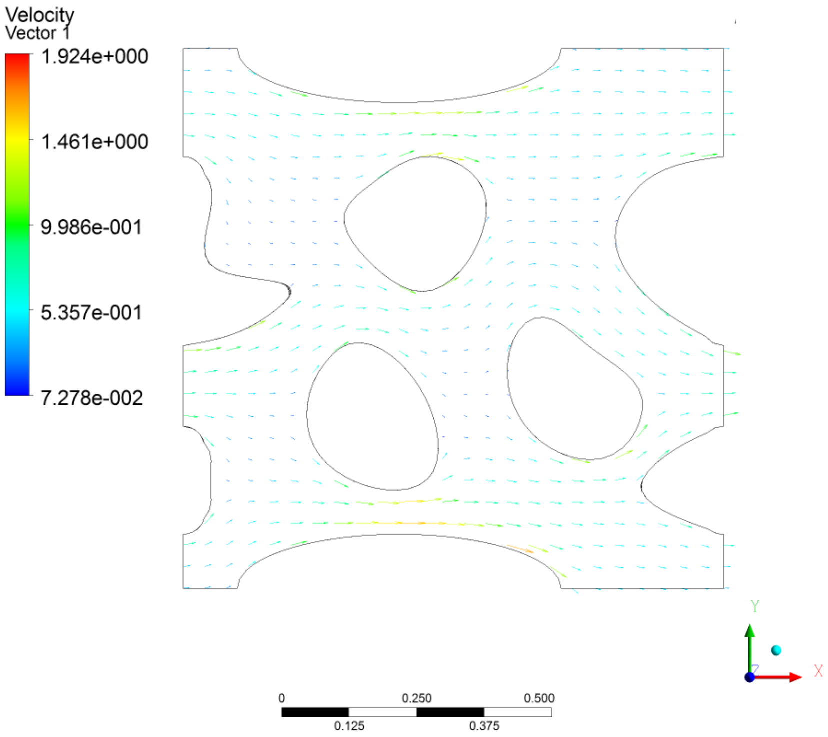
Fig.3. Distributions of (a) k and (b) ε.

The Mean Velocity and Pressure

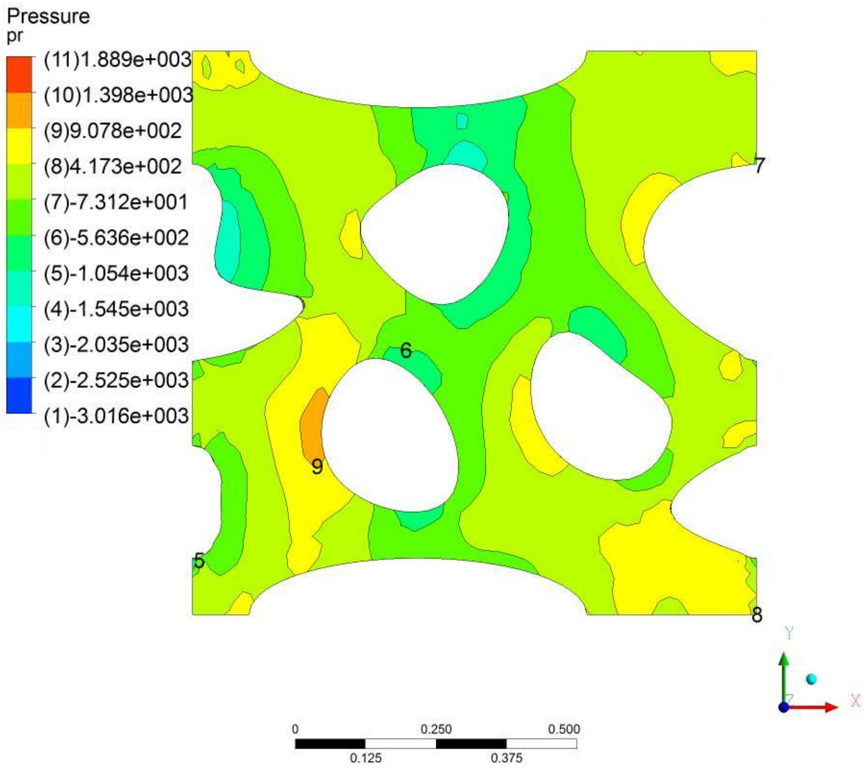
The distributions of the mean velocity and the pressure are shown in Fig. 4(a) and (b).

The mean velocity distribution appears to be quite dominantly aligned in the direction of the imposed pressure gradient

The pressure distribution shows varying patterns along the main flow paths with larger values in front of the solid grain boundaries that are facing the dominant flow direction.



(a)



(b)

Fig.4. Distributions of (a) velocity and (b) pressure.

The Effective Permeability

From the calculations the following are determined:

Porosity: 0.65517
Reynolds number: 3.7E+06
Average velocity (*x*-dirextion): 0.36138
Average velocity (*y*-dirextion): -0.01774

It shows that the flow is clearly turbulent because of the large Reynolds number. The average velocity in *x*-direction, which is obtained by integrating the velocity in *x*-direction, since the external pressure gradient has been imposed in that direction, is the effective permeability of the medium(an aggregate of solid grains). On the other hand the *y*-dirextion average velocity is non-zero, although it is very small, due to the anisotropy of the aggregate. Of course, if there are many grains in the aggregate, it will diminish.

CONCLUSIONS

From the calculations of the turbulent flow in a porous medium with arbitrarily oriented irregular solid grains in a microcell, the following conclusions are drawn.

1. The effective permeability of soil aggregate for turbulent flow can be determined by using the k-ε model. It is dominantly in the longitudinal direction of external pressure gradient.
2. Although the transverse permeability normal to the direction of the external pressure gradient is non-zero, it is small and originates from the anisotropy of the medium
3. The distribution of the turbulent kinetic energy k is larger in the central zones of the pores whereas that of the turbulent dissipation ε is quite uniform.
4. It is worthwhile to extend the computational procedure in this study to medium structures with many grains and totally random distribution patterns.

REFERENCES

1. Pope, S.B., *Turbulent Flows*, Cambridge University Press (2004).
2. ANSYS, Simulation Package, ANSYS Inc. (2010).

ACKNOWLEDGEMENT

This research was supported by the National Research Foundation of Korea (Grant: NRF-2010-0004808) funded by the Ministry of Education, Science and Technology. The financial support is gratefully acknowledged.