Calculation of Permeability for Porous Media – 15108

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ABSTRACT

Creeping flow of a viscous fluid is considered to calculate the permeability of a soil aggregate. Under the assumption of periodicity a unit cell which contains solid grains of irregular geometry is chosen on the microscale and the flow field is calculated by solving a certain boundary-value problem (BVP) known as; Stokes problem. The BVP is obtained by applying the theory of homogenization to slow viscous flow through porous structure on the microscale. It is shown that the permeability is dominantly along the direction of the external pressure gradient on the macroscale with non-vanishing transverse permeability due to anisotropic feature of the grain arrangement.

INTRODUCTION

The subsurface environment, when it is characterized as granular medium, is composed typically of irregularly shaped solid grains with the void space is occupied by either a liquid or a gas or by both in the case of partial saturation. In this work, attention is focused on porous media with full saturation by a liquid.

When there is an externally imposed pressure gradient over the macroscale, the fluid in the pores is driven to flow through the medium. It is a process in which the pressure gradient and the viscous drag balance with each other. From the viewpoint of reservoir management, it is essential to estimate the permeability of the medium. For effective management of the underground repository it is essentially important to know the flow characteristics which may lead to the determination (or reliable estimation) of the medium permeability. Knowledge of flow characteristics will also enable us to readily investigate the transport of contaminant released in the medium.

In this study, a medium which is composed of irregularly shaped solid grains is chosen and the flow field in the pore space is calculated from which the permeability is determined. The calculations are carried out by using CFX and FLUENT packed in ANSYS. The theoretical framework is based on the homogenization theory which systematically combines the processes on the microscale and deduces the governing equations and the effective coefficients on the macroscale [1]. Under two basic assumptions, (i) the periodicity of the medium structure on the microscale with periodic length l and (ii) the periodicity of all variables and material properties over the same length. It is remarked that the periodicity assumption is not very restrictive because the distributions and arrangements over the periodic length are quite arbitrary.

It is shown that, when there is an externally imposed pressure gradient in the x-direction, both the longitudinal permeability (in the x-direction) and the transverse permeability (in the y-direction) appear although the longitudinal one dominates over the transverse one. It is due to anisotropic nature of the solid grain arrangement inside the unit cell, a medium structure which is repeated in both x- and y-directions indefinitely. This implies that information on the grain distribution in a porous medium is very important when reliable and accurate estimation of permeability is sought.

It is emphasized that the approach in the present study does not assume any ad hoc or phenomenological assumptions, except the periodicity assumption which is in a sense not restrictive, and starts from the basic governing laws on the microscale and deduces the effective relations on the macroscale systematically by using the multiple-scale perturbation procedure. The permeability is calculated, not estimated, from the solution to the Stokes problem defined in the pore space.

THE GOVERNING RELATIONS ON THE MICROSCALE

The porous medium is assumed to be composed of the solid grains (Ωs) and the fluid phase (Ω_f) which is saturated by a liquid. The microscale cell domain is represented by $\Omega = \Omega s + \Omega_f$. Each phase is assumed to be connected throughout the porous medium. Fluid flow is induced by a pressure gradient on the macroscale imposed over the medium.

The basic governing relations and the boundary conditions that must be satisfied in the fluid domain Ω_f are desribed without showing the explicit forms..

The governing equations for the fluid on the microscale in the fluid phase (Ω_f) are the conservation laws of mass and momentum[1].

On the boundary Γ between the solid and fluid regions, the liquid velocity vanishes

The governing equations and the boundary conditions are normalized by using the representative scales. It is assumed the inertial effects are small, i.e., the Reynolds number is small.

MULTIPLE SCALE ANALYSIS

The distinguishing features of the multiple scale perturbation analysis are briefly summarized. Recognizing the scale disparity in the process of fluid flow through a porous medium, two distinct length scales are introduced: the microscale (the fast scale which is equivalent to the representative elementary volume in the traditional treatment of the process) and the macroscale (the scale over which the processes of interest take place from the viewpoint of reservoir engineering and management).

The variables are expanded as perturbation series in the following small parameter

$$\frac{\ell}{\ell'} = \epsilon \ll 1 \quad \text{(Eq. 1)}$$

in which ℓ is the microscale length and ℓ' is the macroscale length. Upon expansion of the governing equations and boundary conditions, the microscale boundary-value problems are investigated separately according to the respective order of ϵ and, through volume-averaging over the micro-cell, the macroscale governing equations and the effective coefficients are deduced.

In the process of the multiple scale analysis, a canonical micro-cell boundary-value problem is defined whose solution is used in the calculation of the effective medium properties (effective macroscale coefficients) by averaging over the micro-cell volume.

THE BOUNDARY-VALUE PROBLEM IN THE UNIT CELL ON THE MICROSCALE

For a porous medium with periodic arrays of unit cells a boundary-value problem is defined during the process of applying the multiple-scale expansions to the basic governing relations on the microscale. Specifically the fluid pressure at the leading order is shown to be independent of the microscale. The fluid velocity in the pore and the correction for pressure are then represented in terms of the macroscale pressure gradient as[2]

$$\begin{aligned} \mathbf{v}^{(0)} &= -\mathbf{K} \cdot \nabla' p^{(0)} \\ p^{(1)} &= -\mathbf{S} \cdot \nabla' p^{(0)} \end{aligned}$$

(Eq. 2a, b)

in which dimensionless variables are used and the primed gradient operator is with respect to the macroscale. The numbers in the parentheses are used to denote the order in the perturbation expansion.

For fluid flow the following Stokes problem in dimensionless variables is defined:

$$\begin{aligned} \nabla^2\mathbf{K} - \nabla\mathbf{S} + \mathbf{I} &= 0 \quad \text{in} \quad \Omega_f \\ \nabla \cdot \mathbf{K} &= 0 \quad \text{in} \quad \Omega_f \\ \mathbf{K} &= 0 \quad \text{on} \quad \Gamma \\ \langle \mathbf{S} \rangle &= 0 \\ \mathbf{K} \text{ and } \mathbf{S} \text{ are} \quad & \Omega- \quad \text{periodic.} \end{aligned}$$

(Eq. 3a-e)

In the above, **K**=K_{ij} and **S**=S_j are the fluid velocity in the i-th direction and the fluid pressure variation in the micro-cell due to externally imposed pressure gradient in the j-th direction. The unprimed gradient operator is with respect to the microscale coordinates. The pair of angle brackets in (Eq. 3d) is the volume average over Ω as defined below in (Eq. 4).

The momentum conservation of the fluid with no convective inertia is given in (Eq. 3a) in which a unit force drives the flow. The continuity equation is given in (Eq. 3b) and the no-slip condition on the fluid-solid interface is given in (Eq. 3c). Equation (Eq. 3d) is imposed to ensure the uniqueness of the fluid pressure. Lastly (Eq. 3e) is imposed to satisfy the periodicity condition.

The macroscale permeability tensor of rank two is then given by the micro-cell volume average of **K** as

$$\langle \mathbf{K} \rangle = \frac{1}{\Omega} \int_{\Omega} \mathbf{K} d\Omega$$

(Eq. 4)

and the Darcy's law is given as

$$\langle \mathbf{v}^{(0)} \rangle = -\langle \mathbf{K} \rangle \cdot \nabla' p^{(0)}$$

(Eq. 5)

where the left-hand side is the seepage velocity and the primed gradient is the derivative of the fluid pressure over the macroscale. This serves as the momentum equation on the macroscale.

THE MICROCELL GEOMETRY, DISCRETIZATION, AND NUMERICAL SOLVER

The geometry of the unit cell on the microscale is as shown in Fig. 1 in which six irregularly shaped solid grains are shown. Three particles located across the boundary are reappearing after the microscale length ℓ: two irregular shaped grains along the horizontal direction and one elliptical shape in the vertical direction. The solid grains are shown in empty shapes and the pore space is shown with discretized mesh(which is one of the meshes used in solving the Stokes problem numerically).

Except for simple and elementary geometries the Stokes problem does not allow analytic solution and has to be solved numerically. For this purpose the commercial software FLUENT and CFX for flow analysis (packed together with other purpose ones in ANSYS) have been used.

In solving the Stokes problem, six progressively finer finite element meshes were used. They are labelled as Mesh 10, 20, 30, 40, 50, and 60 respectively. The number in the mesh label denotes the number of elements along the horizontal and vertical sides of the unit cell. As the number increases, the number of finite elements increases sharply in proportion to the square of the label number.

Although FLUENT and CFX showed some discrepancy in the results for coarse meshes, as the mesh are refined, the results from the two software resources became very close to each other. For example, the medium permeability values calculated by FLUENT and CFX showed an error of 2% or less. In what follows, the calculations obtained by CFX will mainly be discussed.

Two other meshes(Meshes 40 and 60) finer than the one in Fig. 1, are shown in Fig.2.

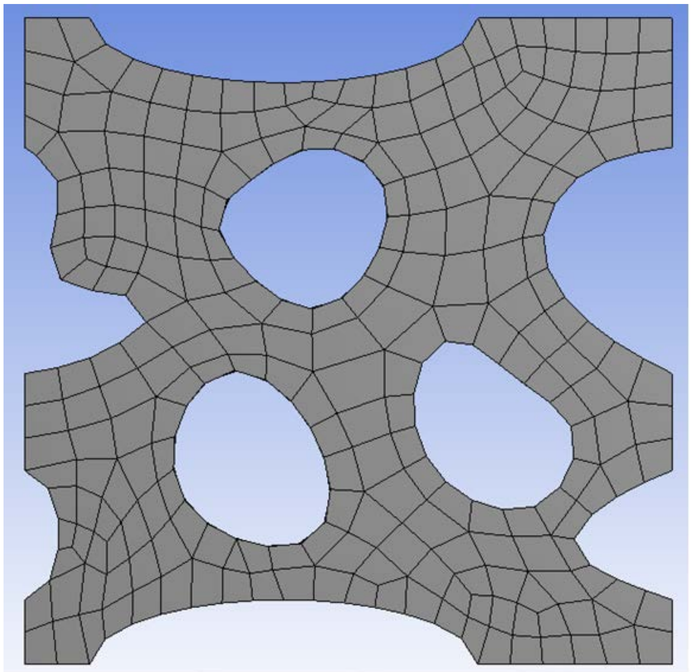
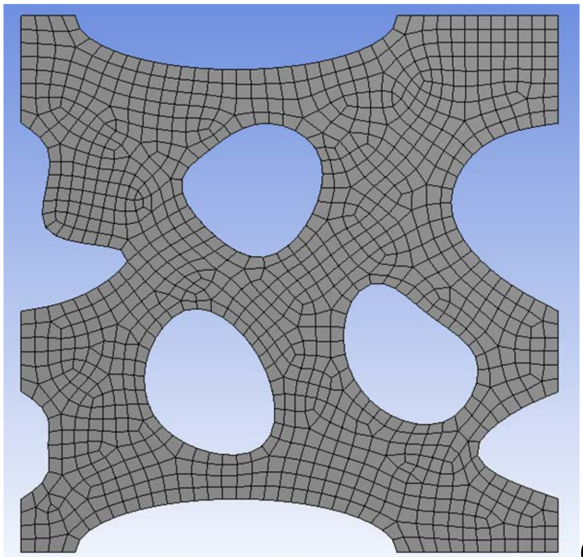
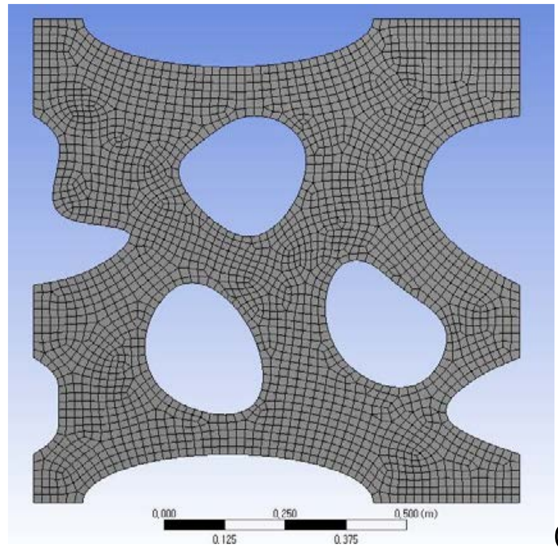


Fig.1. A finite element mesh used in solving the Stokes problem(Mesh 20).


(a)



(b)

Fig.2. (a) Mesh 40 and (b) Mesh 60 (finer than Mesh 20) used in solving the Stokes problem.

THE FLUID VELOCITY AND PRESSURE DISTRIBUTIONS IN THE PORES

The velocity distribution in the pores determined from the finest mesh(Mesh 60 shown in Fig. 2(b)) is shown in Fig. 3 which has been adjusted automatically by CFX for clear and convenient display of the velocity arrows by reducing the number of arrows in the plot.

Due to the external pressure gradient in the x-direction the velocity vestors are aligned predominantly in the x-direction. Notice that, due to the periodicity condition, (Eq. 3e), the velocity arrows on the left boundary are repeated on the corresponding portions of the right boundary. Of course, the same periodicity patterns are maintained between the lower and upper boundaries. Due to the no-slip condition for the velocity, (Eq. 3c), the fluid velocity vanishes on the surface of the grains.

The pressure distribution in the pore space is shown in Fig. 4. Notice that the pressure contour lines are quite vertical in the relatively large pore spaces which is consistent with the direction of the external pressure gradient(x-dir). On the other hand, in the channel-like narrow pores near the central region along x-direction the pressure changes along the channel direction with relatively steep variation where, in fact, the velocity takes the largest values as seen in Fig. 3.

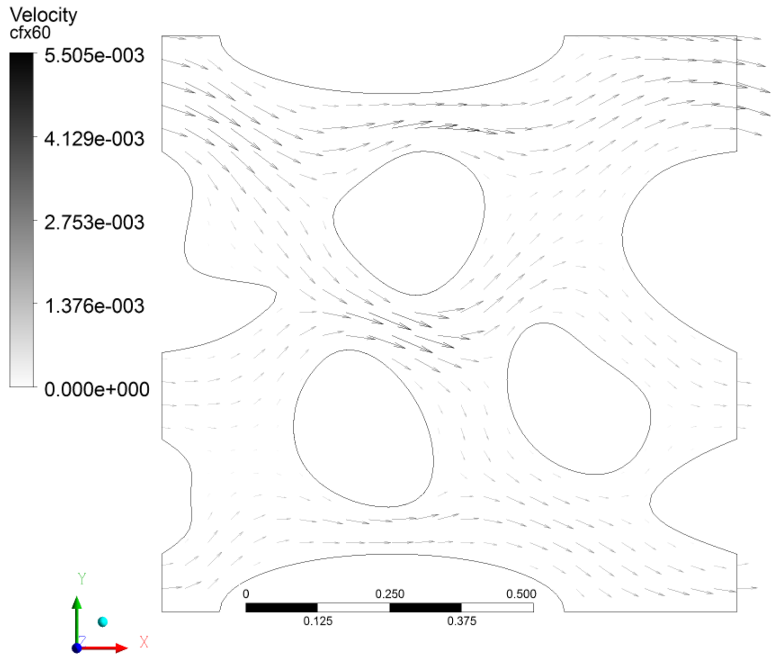


Fig. 3. Velocity distribution in the pores (from Mesh 60) when the external pressure gradient is in the x-direction.

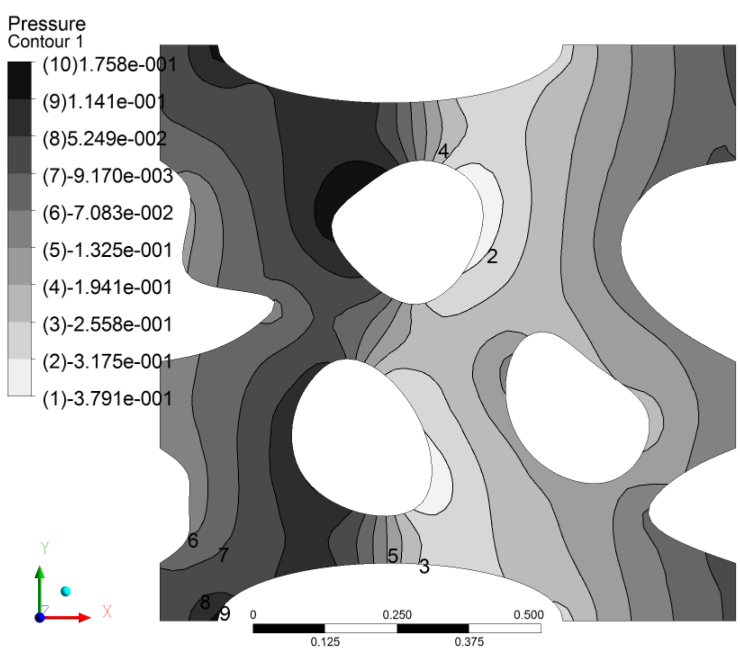


Fig. 4. Pressure distribution in the pores when the external pressure gradient is in the x-direction.

RESULTS AND DISCUSSION ON THE PERMEABILITY

From the velocity field the permeability is determined by using (Eq. 4), i.e., by taking the volume average over the micro-cells (Meshes 20, 40, and 60 have been shown in Fig. 1 and Fig. 2.)

The values calculated from the six different meshes (Meshes 10 through 60) in CFX are summarized in Table 1 below. The results obtained from FLUENT are also summarized in Table 2 for comparison purpose. The same meshes were used in both.

Table 1 Permeability values from different meshes (CFX).

Mesh	<Kxx>	<Kyx>	Porosity	Reynolds No.
10	3.3648 E-03	-1.1507 E-04	0.6836	4.77 E-03
20	1.5259 E-03	-1.7774 E-04	0.6624	2.31 E-03
30	1.2490 E-03	-1.4409 E-04	0.6582	2.02 E-03
40	1.1359 E-03	-1.3416 E-04	0.6569	1.90 E-03
50	1.0768 E-03	-1.3309 E-04	0.6561	1.84 E-03
60	1.0475 E-03	-1.2846 E-04	0.6558	1.80 E-03

Table 2 Permeability values from different meshes (FLUENT).

Mesh	<Kxx>	<Kyx>	Porosity	Reynolds No.
10	2.5314 E-03	-1.8000 E-04	0.6834	8.72 E-04
20	1.3943 E-03	-1.4858 E-04	0.6624	2.73 E-04
30	1.1885 E-03	-1.4049 E-04	0.6582	1.74 E-04
40	1.1006 E-03	-1.3216 E-04	0.6569	1.40 E-04
50	1.0537 E-03	-1.2993 E-04	0.6561	1.01 E-04
60	1.0255 E-03	-1.2814 E-04	0.6558	8.71 E-05

The permeability <Kxx>, the permeability in the x-direction due to the macroscale pressure gradient in the x-direction, is about eight times larger than <Kyx>, the permeability in the y-direction due to the macroscale pressure gradient in the x-direction. In other words, the longitudinal permeability dominates over the transverse permeability. This is reasonable in the sense that the solid grains are of granular shape with an elongated one at the top(or bottom) boundary. It should be noted that the transverse permeability is not zero since the medium structure is not isotropic. In real natural media, the same kind of anisotropy is therefore expected because the grains in the underground environment are mostly irregularly shaped to a higher degree.

However, if the number of grains in a unit cell increases, isotropic behavior of the permeability is possible for totally random many-particle unit cell structure. It is expected that the computation becomes quite demanding and formidable.

It is also observed that the longitudinal permeability is positive whereas the transverse permeability is negative. It is the consequence of the pattern of solid grain arrangement inside the micro-cell. This implies that the information on both the shape and the orientation of particles is essentially important when one tries to calculate the medium permeability with satisfactory level of confidence.

The porosity value converges quickly as the mesh is refined, i.e., as the grain boundaries become smooth with the refinement of discretization. In principle, there is no distinction between CFX and FLUENT because the geometry is simply the same in both CFX and FLUENT.

The results by CFX and FLUENT show very small difference (about 2%) at the finest mesh (Mesh 60). Since satisfactory agreement between the two has been made, no further refinement of the mesh has been pursued.

In the flows governed by the Stokes problem defined in (Eq. 3), the inertial effects are absent. Accordingly the flow calculations have been carried out by laminar flow. For the purpose checking, the Reynolds number has been determined in each calculation. The values are summarized in Tables 1 and 2. In fact, the Reynolds number is very small and it is confirmed that the convective inertia has been suppressed in the calculations.

CONCLUSIONS

From the calculations of the permeability in a porous medium with arbitrarily oriented irregular solid grains in a unit cell on the microscale the following conclusions are drawn.

1. The permeability of soil aggregate strongly depends on and is influenced by the distribution pattern of the solid grains in the microcell in the medium.
2. The transverse permeability (permeability in the direction normal to the direction of the external pressure gradient) is in general not zero although its magnitude is much smaller than the longitudinal one.
3. Calculation of permeability for a porous medium with irregularly shaped grains is carried out efficiently by using finite elements.
4. It is worthwhile extending the computational procedure in this study to medium structures with many grains of various shapes and totally random distribution patterns.

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