Permeability and Dispersion Coefficients in Rocks with Fracture Network - 14156

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ABSTRACT

Fluid flow and solute transport are considered for a rock medium with a fracture network with regard to the effective permeability and the dispersion coefficients. To investigate the effects of individual fractures a three-fracture system is chosen in which two are parallel and the third one connects the two at different angles. Specifically the micro-cell boundary-value problems (defined through multiple scale analysis) are solved numerically by using finite elements to calculate the permeability and dispersion coefficients. It is shown that the permeability depends significantly on the pattern of the fracture distribution and the dispersion coefficient is influenced by both the externally imposed pressure gradient (which also reflects the flow field) and the direction of the gradient of solute concentration on the macroscale.

INTRODUCTION

Rock media are in many cases characterized by the existence of fracture network in which individual fractures are aligned in various directions. The fluid flow in a rock medium with fracture network driven by externally imposed pressure gradient is therefore dependent on the fracture arrangement in the network. The dependence of the flow through fractured medium is characterized by the effective permeability of the medium (usually known as the hydraulic conductivity and is not equal to the intrinsic permeability which depends only on the geometric and fab[1]rication properties of the medium.)

Solute matter released in the medium is transported through fracture network by existing flow field. The spreading of solute is influenced by both the molecular diffusion and hydrodynamic dispersion (consequence of Taylor dispersion due to non-uniform fluid velocity distribution in the fracture). The spreading pattern is also strongly affected by the arrangement of the fractures.

For effective management of the underground repository located in a rock medium with fracture network it is important to evaluate the characteristics of both the permeability and the solute transport. In this study, the process of fluid flow and solute transport through a macroscale medium with fracture network is investigated with emphasis on the effective macroscale coefficients: the permeability and the dispersion coefficients.

The theoretical framework is based on the homogenization theory which systematically combines the processes on the microscale and deduces the governing equations and the effective coefficients on the macroscale [1]. Under two basic assumptions, (i) the periodicity of the medium structure on the

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microscale with periodic length l and (ii) the periodicity of all variables and material properties. The periodicity assumption is not very restrictive because the distributions and arrangements over the periodic length are quite arbitrary.

The fracture network used in the present study is composed of three fractures. Two fractures are parallel to each other and are aligned along 45-deg counterclockwise from the horizontal direction. The third one connects the parallel fractures at different angles. Specifically three cases are considered: (i) the connecting fracture is normal to the parallel fractures (normal), (ii) the connecting fracture is nearly horizontal (forward), and (iii) the connecting fracture is nearly vertical (backward).

THE GOVERNING RELATIONS ON THE MICROSCALE

The porous medium is assumed to be composed of the matrix (Ωs) of solid rock phase and the fluid phase (Ω_f) which constitute the fracture network. Each phase is assumed to be connected throughout the porous medium. Fluid flow is induced by macroscopically imposed pressure gradient over the medium.

Without display of explicit formulas the basic governing relations and the boundary conditions that must be satisfied in the fluid domain Ω_f are desribed.

The basic governing equations on the microscale in the fluid phase (Ω_f) are the conservation of mass, the conservation of momentum and the conservation of solute matter[1].

On the boundary Γ between the solid and fluid, the liquid velocity vanishes and the solute flux along the normal direction should vanish.

The governing equations and the boundary conditions are normalized by using the representative scales (refer to [1] for details). An important dimensionless parameter appears in the analysis of solute transport: the Pecet number that signifies the relative importance of the convective solute transfer to the diffusive solute transfer.

MULTIPLE SCALE ANALYSIS

The distinguishing features of the multiple scale perturbation analysis are briefly summarized. Recognizing the scale disparity in the process of fluid flow and solute transport, two distinct length scales are introduced: the microscale (the fast scale which is equivalent to the representative elementary volume in the traditional treatment of the process) and the macroscale (the scale over which the processes of interest take place from the viewpoint of reservoir engineering and management).

The variables are expanded as perturbation series in the following small parameter

$$\frac{\ell}{\ell'} = \epsilon \ll 1 \qquad \text{(Eq. 1)}$$

in which l is the microscale length and l' the macroscale length. Upon expansion of the governing equations and boundary conditions, the microscale boundary-value problems are investigated separately according to the respective order of ϵ and, through volume-averaging over the micro-cell, the effective macroscale governign equations are derived.

In the process of the multiple scale analysis, several canonical micro-cell boundary-value problems are defined whose solutions are used in the calculation of the effective medium properties (effective macroscale coefficients) by averaging over the micro-cell volume.

THE MICRO-CELL BOUNDARY-VALUE PROBLEMS

(1) For fluid flow the following Stokes problem in dimensionless variables is defined:

$$\begin{array}{rcll} \nabla^2\mathbf{K} - \nabla\mathbf{S} + \mathbf{I} & = & 0 & \text{in} \quad \Omega_f \\ \nabla\cdot\mathbf{K} & = & 0 & \text{in} \quad \Omega_f \\ \mathbf{K} & = & 0 & \text{on} \quad \Gamma \\ \langle\mathbf{S}\rangle & = & 0 & \\ \mathbf{K} \text{ and } \mathbf{S} \text{ are} & \Omega- & \text{periodic.} & \end{array}$$

(Eq. 2)

In the above, $\mathbf{K}=K_{ij}$ and $\mathbf{S}=S_j$ are the fluid velocity in the i-th direction and the fluid pressure variation in the micro-cell due to externally imposed pressure gradient in the j-th direction.

The macroscale permeability tensor of rank two is then given by the micro-cell volume average of **K** as

$$\langle\mathbf{K}\rangle = \frac{1}{\Omega}\int_{\Omega}\mathbf{K}d\Omega$$

(Eq. 3)

and the Darcy's law is given as

$$\langle\mathbf{v}^{(0)}\rangle = -\langle\mathbf{K}\rangle\cdot\nabla' p^{(0)}$$

(Eq. 4)

where the left-hand side is the seepage velocity and the primed gradient is the derivative of the fluid pressure over the macroscale. This serves as the momentum equation on the macroscale.

(2) The solute transport process gives, in the process of multiple scale analysis, the following boundary-value problem:

$$\begin{array}{rll} Pe\left(\tilde{\mathbf{v}}^{(0)} - \mathbf{v}^{(0)}\cdot\nabla\mathbf{M}\right) = \nabla^2\mathbf{M} & \text{in} & \Omega_f \\ (\mathbf{I}-\nabla\mathbf{M})\cdot\mathbf{N} = 0 & \text{on} & \Gamma \\ \mathbf{M} \text{ is} \quad \Omega- & \text{periodic.} & \\ <\mathbf{M}>= 0 & & \end{array}$$

(Eq. 5)

where $\tilde{\mathbf{v}}^{(0)}$ is the fluid velocity fluctuation about its mean over the micro-cell average, Pe is the Peclet number that signifies the importance of the convective solute transfer relative to the diffusive solute transfer, and $\mathbf{M}=M_j$ is the solute concentration in the fracture relative to the average concentration caused by the macroscale concentration gradient in the j-th direction.

The effective macroscale dispersion tensor is given by the following volume average

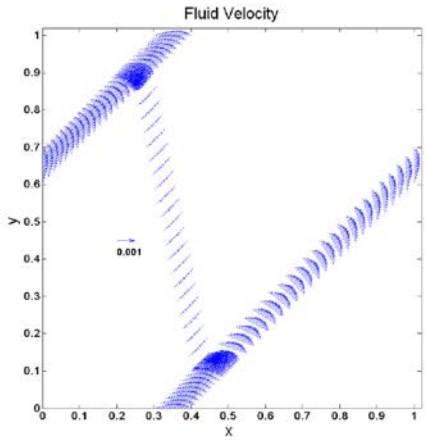
$$\mathbf{D}' = D_{ij} = \left\langle \frac{\partial M_i}{\partial x_k}\frac{\partial M_j}{\partial x_k}\right\rangle - \left\langle \frac{\partial M_i}{\partial x_j} + \frac{\partial M_j}{\partial x_i}\right\rangle + n'\delta_{ij}$$

(Eq. 6)

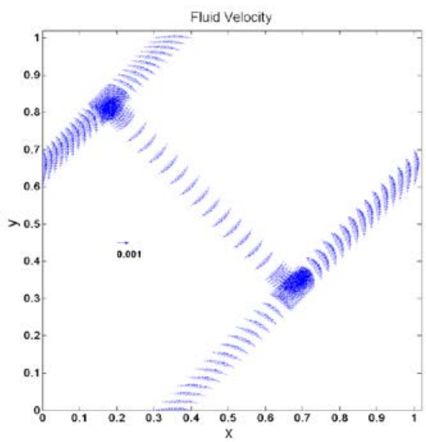
where *n'* is the porosity of the medium, δ_{ij} is the Kronecker delta, and summation over the repeated index *k* is assumed.

THE FRACTURE GEOMETRIES AND NUMERICAL CALCULATION

(1) The geometries of fracture network used in the present study to calculate the permeability and the dispersion coefficients are shown in Fig. 1 (a) – (c) and in Fig. 2 (a) – (c) [the geometries of Fig. 1 and 2 are identical] in which the velocity fields are also shown.



(a)



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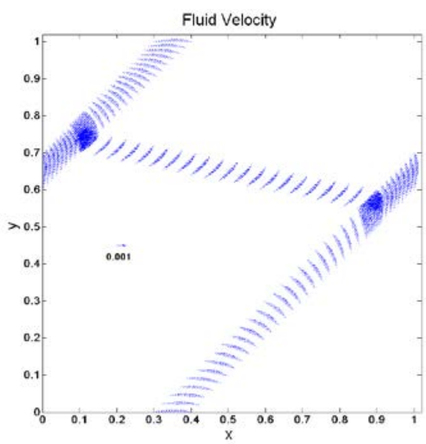
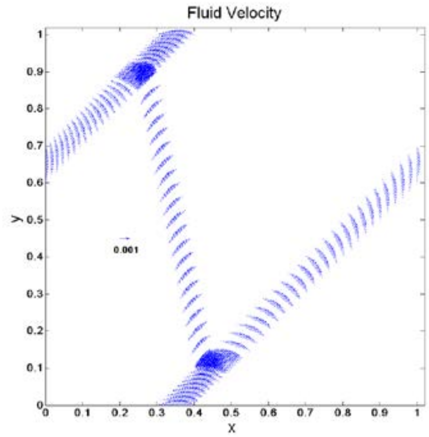
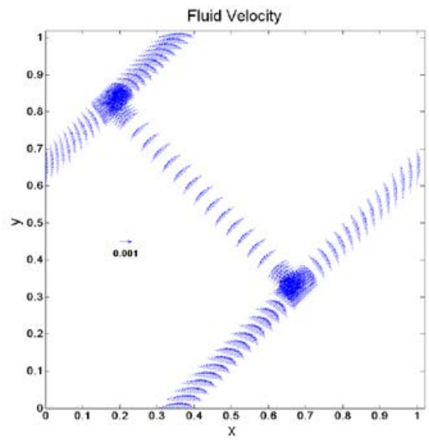


(c)

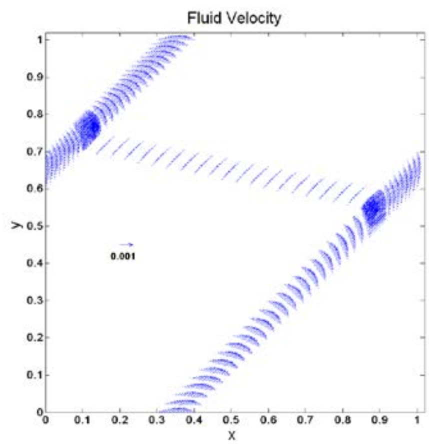
Fig. 1. Three different types of fracture network: (a) Backward, (b) Normal, and (c) Forward. The macroscale pressure gradient is in the horizontal (x-) direction.



(a)



(b)



(c)

Fig. 2. Three different types of fracture network: (a) Backward, (b) Normal, and (c) Forward. The macroscale pressure gradient is in the vertical (y-) direction.

RESULTS AND DISCUSSION

Two micro-cell boundary-value problems defined above, i.e., the Stokes problem for flow field and the convective diffusion problem for solute concentration were solved by using two-dimensional finite elements. Specifically quadratic basis function and linear basis function were used for the Stokes problem and quadratic basis function was used for the calculation of Mx and My in the convective diffusion problem. The details of numerical implementation are omitted.

Flow field in the fracture network

The results of flow field calculation are shown in Fig. 1 for the case of the macroscale pressure gradient in the horizontal (x-) direction and Fig. 2 for macroscale pressure gradient in the vertical (y-) direction. Because of the difference of the macroscale pressure gradient the flow direction in the third (connecting) fracture is from the left to the right in the case of the horizontal (x-) direction pressure gradient and is from the right to the left in the case of the vertical (y-) direction pressure gradient. The flow pattern in the parallel fractures is from the left to the right in both cases which conform with the pressure gradient direction.

Various meshes were tested to examine the convergence of the permeability (volume-averaged flow intensity defined above) with primary emphasis on the effect of refining discretization across the fracture. The results for the case of horizontal (x-) direction macroscale pressure gradient are summarized in Table I in which the convergence pattern is displayed for progressively refined spacing (Ny increasing).

Table I. Convergence pattern of the effective permeability for three different types of fracture networks.

Three Fractures (backward)				
Ny	Kxx	Kyx	Kyy	Kxy
2	2.2103E-05	1.8536E-05	3.3769E-05	1.8536E-05
4	2.2069E-05	1.8538E-05	3.3819E-05	1.8538E-05
6	2.2040E-05	1.8531E-05	3.3813E-05	1.8531E-05
8	2.2017E-05	1.8526E-05	3.3803E-05	1.8526E-05

Three Fractures (normal)				
Ny	Kxx	Kyx	Kyy	Kxy
2	2.8434E-05	1.4172E-05	2.8491E-05	1.4172E-05
4	2.8507E-05	1.4190E-05	2.8541E-05	1.4190E-05
6	2.8522E-05	1.4192E-05	2.8546E-05	1.4192E-05
8	2.8526E-05	1.4192E-05	2.8545E-05	1.4192E-05

Three Fractures (forward)				
Ny	Kxx	Kyx	Kyy	Kxy
2	3.3653E-05	1.8478E-05	2.2140E-05	1.8478E-05
4	3.3787E-05	1.8509E-05	2.2093E-05	1.8509E-05
6	3.3796E-05	1.8510E-05	2.2062E-05	1.8510E-05
8	3.3790E-05	1.8509E-05	2.2037E-05	1.8509E-05

The reason for examining the convergence with Ny is that the velocity distribution shows significant variation across the fracture. It is seen that the convergence is surprisingly satisfactory when Ny becomes 8 (i.e., there are 8 node spacings (9 nodes) across each fracture).

The permeability <Kxx>, when the pressure gradient is in the horizontal (x-) direction, increases as the connecting fracture changes from the backward through normal to the forward because the connecting fracture is more and more aligned with the pressure gradient direction. On the other hand, the permeability <Kyx> does not show any consistent trend with varying orientation of the connecting fracture.

The permeability <Kyy>, when the pressure gradient is in the vertical (y-) direction, decreases as the connecting fracture changes in the same fashion as above since the blockage effect of the connecting fracture increases. As in the case of horizontal (x-) direction pressure gradient, <Kxy> does not show any consistent trend.

However, it is noted that the symmetry relation <Kxy> = <Kyx> is always satisfied.

Mx and My fields in the fracture network

The distribution of scalar field Mx and My is displayed in Fig 3 in the case of horizontal(x-) direction pressure gradient ($\theta=0°$) for various Peclet numbers(Pe) ranging from 0 (pure diffusion case) upto 100. As Pe increases the extent of variation of Mx and My becomes larger indicating that the dispersion tensor increases.

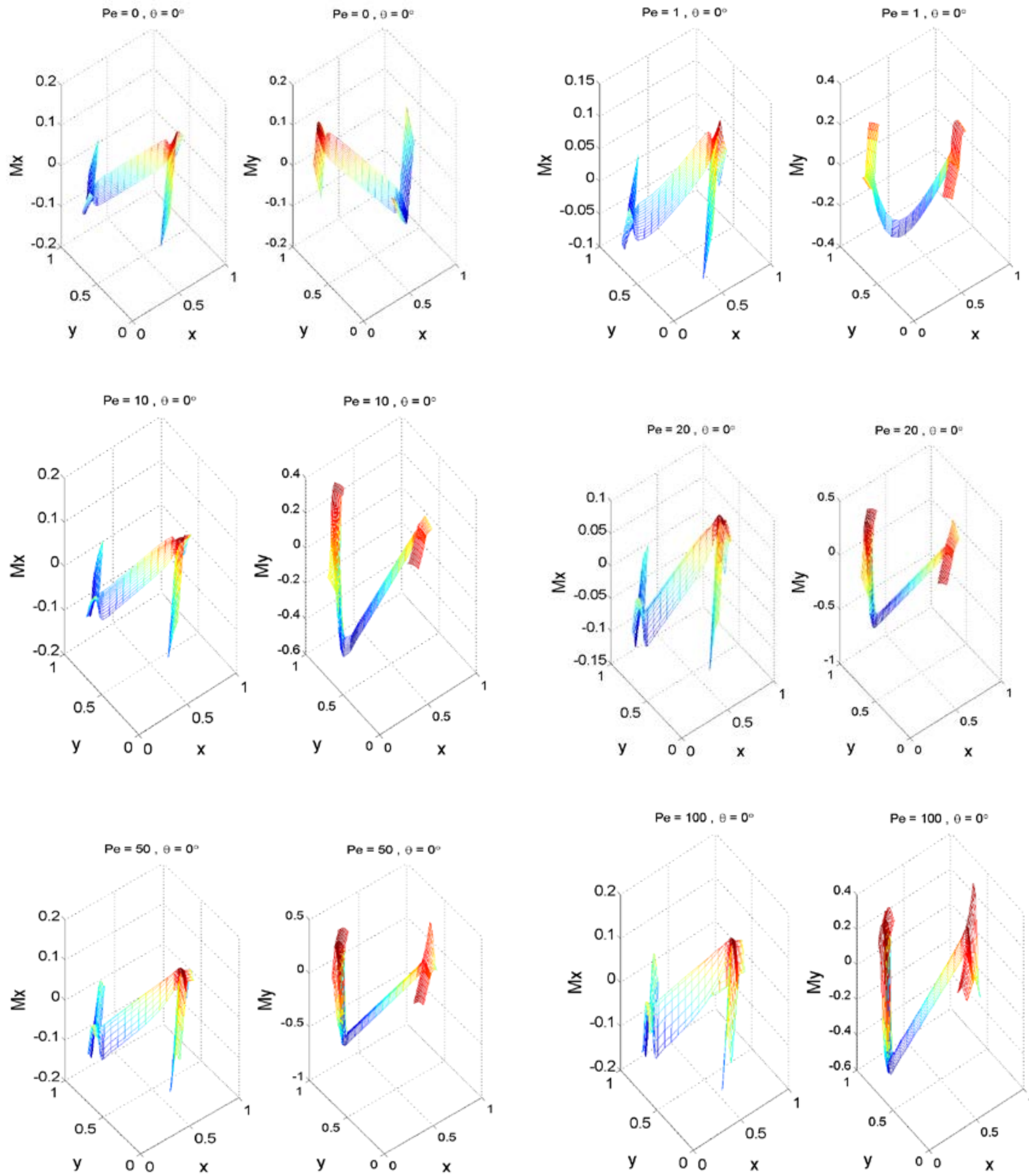


Fig. 3. Distributions of Mx and My for normal fracture (the connecting fracture is normal to the parallel fractures) for Pe (Peclet number)=0, 1, 10, 20, 50, 100.

Dxx for arbitrary pressure gradient direction over the fracture network

Calculated dispersion coefficient Dxx for the case of normal fracture, the dispersion coefficient in the x-direction when the macroscale concentration gradient is the x-direction, is shown in Fig. 4 for various directions of the macroscale pressure gradient. The angle θ changes from 0° (x-direction pressure gradient) to 150° counterclockwise.

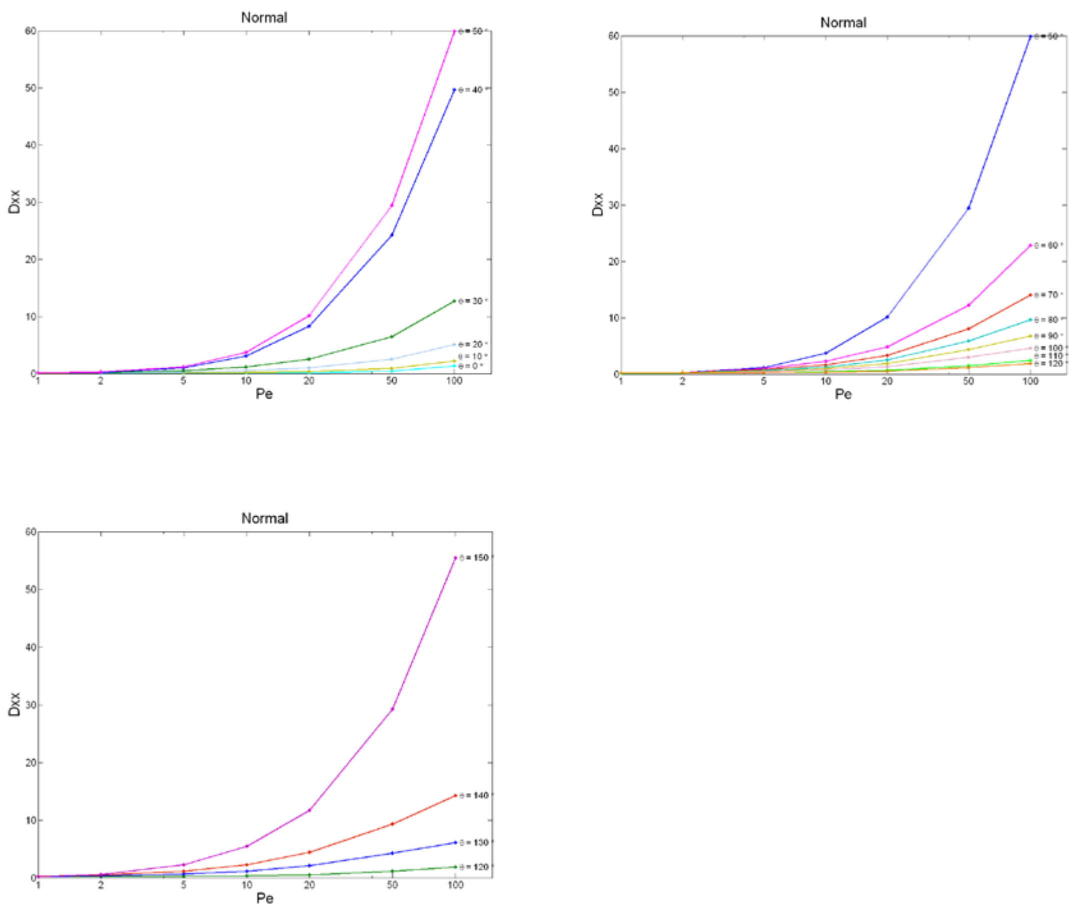


Fig. 4. Variation of Dxx with Peclet number for various directions of the macroscale pressure gradient. The angle θ is measured from the horizontal direction counterclockwise.

Clearly Dxx increases with Pe since the importance of convection increases with Pe which means that, for the same molecular diffusion coefficient and medium geometry, Pe increases with the flow intensity thereby enhancing the velocity gradient across the fractures and increased spreading of solute matter in the medium.

However because of the alignment of the fractures the variation of Dxx, in this case, increases with θ upto θ = 50° and then decreases in the range of θ = 50° and θ = 120° and then increases again with θ. This implies that the variation of Dxx with θ is strongly dependent on the distribution of fractures in the medium.

CONCLUSIONS

From the calculations of the permeability and dispersion coefficients for solute in a rock medium with a fracture network the following conclusions are drawn.

1. The permeability of fractured medium depends on the primary orientation of the fracture network and is influenced by the connecting fractures in the medium.
2. The cross permeability, e.g. permeability in the direction normal to the direction of the external pressure gradient, is rather insensitive to the orientation of the fracture network.
3. Calculation of permeability is most efficiently achieved with optimal discretization across individual fractures and is rather insensitive to the discretization along the fracture.
4. The longitudinal dispersion coefficient Dxx of a fractured medium depends on both the macroscale concentration gradient and the direction of the flow (pressure gradient). Hence both features must be considered when investigating solute transport in a fractured medium.

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REFERENCE

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