Mann-Kendall Test for Analysis of Groundwater Contaminant Plume Stability and Evaluation of Sampling Frequency for Long-Term Monitoring – 13233

Jeffrey R. Walker and Toby R. Harrison
* Elvado Environmental LLC, 9724 Kingston Pike, Suite 603, Knoxville, TN 37922, jwalker@elvadollc.com

ABSTRACT

This paper describes a spreadsheet-based approach for applying the Mann-Kendall (MK) Test to identify statistically significant increasing or decreasing concentration trends, stable concentration trends (not increasing or decreasing), and indeterminate concentration trends (no trend) defined by time-series groundwater monitoring data for inorganic, organic, or radiological contaminants. The approach has been applied in support of ongoing long-term monitoring (LTM) of groundwater contamination at the U.S. Department of Energy (DOE) Y-12 National Security Complex (Y-12) in Oak Ridge, Tennessee and elsewhere on the DOE Oak Ridge Reservation (ORR), and has proven effective at minimizing subjective bias in the evaluation and interpretation of contaminant concentration trend data. Application of the approach for the purposes of optimizing groundwater sampling frequency for LTM also is outlined.

TECHNICAL APPROACH

The following five-step technical approach was modified from methods described in Gilbert (1987) and Wiedemeier *et al*. (1999) that use the MK Test from Kendall (1975) in the statistical evaluation of time-series environmental monitoring data. For each groundwater sampling location and inorganic, organic, or radiological contaminant:

Step 1: Input contaminant concentration data from a minimum of four and no more than ten independent sampling events arranged in chronological order by sampling date and time.

Step 2: Based on input data from at least four sampling events, calculate the value (positive, negative, or zero) of the MK Test statistic (S). The MK Test quantifies the relative differences (higher, lower, or equal) between each of the time-series sampling results, regardless of magnitude, with $S > 0$ indicating an upward (increasing) trend, $S < 0$ indicating a downward (decreasing) trend, and $S = 0$ indicating no trend over time.

Step 3: Plot the calculated S value for the corresponding number of sampling events on a 90% confidence chart to determine if the S value equals or exceeds the threshold limit, which indicates whether the time-series data delineate a statistically significant trend.

Step 4: Calculate the coefficient of variation (CV) to characterize the relative variability of the time-series sampling results used to determine the corresponding S value for the contaminant.

Step 5: Determine if the input time-series data for the contaminant define a statistically significant increasing ($S>0$) or decreasing ($S<0$) concentration trend, a stable concentration trend (i.e., no statistically significant increase or decrease), or no trend (indeterminate because of high data variability) based on the calculated S value, the plot of the S value on the confidence chart, and the corresponding CV.

Data Input

For the purposes of this evaluation, the MK Test input data for each applicable contaminant must include analytical results from at least four and no than ten sampling events for the sampling location. Analytical results that do not meet applicable data quality objectives should be excluded from the input data. Non-detect results must be replaced with a designated substitute value, such as the practical quantitation limit (or fraction thereof), that is less than the lowest value in the input data.

MK Test

The MK Test can be viewed as a nonparametric test for zero slope of the linear regression of time-oriented data versus time (Gilbert 1987). This approach is especially suitable for quantitative evaluation of environmental data because it is suitable for nonparametric data distributions, accommodates non-detect analytical results, and is based on the relative magnitudes of the data rather than the specific analytical results. The MK Test method is applicable for datasets that contain no more than 40 analytical results, with a normal approximation test of the calculated S value appropriate for larger (i.e., n >40) datasets (Gilbert 1987).

With the input data ordered by date/time (x_1, x_2, …x_n) where x_j is the datum at time j and x_k is the datum at time k (and $j > k$), the MK Test statistic S equals the summed array of the values assigned by the indicator function sgn($x_j - x_k$), which returns a value of 1 if the compared concentration is higher (i.e., $x_j - x_k > 0$), returns a value of 0 if the compared concentration is equal (i.e., $x_j - x_k = 0$), and returns a value of -1 if the compared concentration is lower (i.e., $x_j - x_k < 0$). Table 1 illustrates the approach of calculating the S value from the time-series results for ten sampling events, per the methodology described by Weidemier *et al*. (1999), along with S values calculated from the time-series results for other applicable numbers of sampling events (above the minimum of four), which illustrates temporal changes in the calculated S values.

Table 1. Example input data for worksheet calculation of MK Test statistic.

Sampling Event	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Sampling Date	**1/08**	**7/08**	**1/09**	**7/09**	**1/10**	**7/10**	**1/11**	**7/11**	**1/12**	**7/12**
Sampling Result	**1.1**	**1.2**	**1.2**	**1.2**	**1.3**	**1.2**	**1.2**	**1.3**	**1.3**	**1.2**
Compare to Event 1	.	1	1	1	1	1	1	1	1	1
Compare to Event 2	.	.	0	0	1	0	0	1	1	0
Compare to Event 3	.	.	.	0	1	0	0	1	1	0
Compare to Event 4	.	.	.	.	1	1	-1	1	1	0
Compare to Event 5	.	.	.	.	.	-1	0	0	0	-1
Compare to Event 6	.	.	.	.	.	.	0	1	1	0
Compare to Event 7	.	.	.	.	.	.	.	1	1	0
Compare to Event 8	.	.	.	.	.	.	.	.	0	-1
Compare to Event 9	.	.	.	.	.	.	.	.	.	-1
MK Test Statistic (*S*)				**3**	**7**	**8**	**8**	**14**	**20**	**18**

Data Variability

The CV serves as a simple calculation (CV = standard deviation/mean) intended to gauge the relative variability of the time-series groundwater sampling results used to calculate the corresponding value of the MK Test statistic. The CV is used to identify highly variable time-series data (i.e., CV >1) from which no statistically significant trend can be determined (at the specified confidence level).

Confidence Level Chart

Per the methodology from Weidemier *et al*. (1999), the value of the MT Test statistic *S* calculated from the time-series data for each applicable corresponding number of sampling events is plotted on a 90% confidence level chart. Figure 1 below shows how the *S* values calculated from the example input data on Table 1 plot on the 90% control chart.

Figure 1. 90% confidence level chart for example MK Test input data.

90% CONFIDENCE LEVEL CHART								TREND EVALUATION
***S* Value**	**Number of Sampling Events**							
	4	**5**	**6**	**7**	**8**	**9**	**10**	
0								**NO TREND PRESENT** CV < 1: Stable (not increasing or decreasing) CV > 1: No trend (due to data variability)
± 1								
± 2								
± 3	●							
± 4								
± 5								
± 6								
± 7		●						
± 8								
± 9			●	●				
± 10								
± 11								
± 12								
± 13								
± 14					●			
± 15								
± 16								**TREND PROBABLY PRESENT** S > 1: Upward/increasing trend S < 1: Downward/decreasing trend
± 17								
± 18							●	
± 19								
± 20						●		

The *S* values that plot at or above the applicable threshold limit on the 90% confidence level chart indicate that the associated time-series data probably define a statistically significant upward (*S* > 0) or downward (*S* < 0) trend. The threshold limit for the corresponding number of sampling events is based on <10% probability that the time-series data do not delineate a trend when the calculated *S* value equals or exceeds the specified limit. Higher confidence levels (e.g., 95%) have higher threshold limits based on lower probabilities (e.g., <5%) that the calculated S value does not indicate a statistically significant trend. Gilbert (1987) presents a summary table of MK Test probability values calculated for up to ten sampling events; probability values for up to 40 sampling events are tabulated in Hollander and Wolfe (1973).

APPLICATION OF TECHNICAL APPROACH

The technical approach described in this paper has proven an effective means of evaluating long-term contaminant concentration trends and has been used for multiple purposes of various groundwater monitoring programs at Y-12 and the ORR. For example, findings of the MK Test evaluation of contaminant concentration trends were used to optimize groundwater sampling frequencies for LTM at applicable former waste management sites and operations facilities, as illustrated below in Table 3, based on the assumption that statistically significant concentration trends reflect the relative flux (increasing, decreasing, or unchanged) of contaminants via the groundwater flowpaths monitored by the well and the overall stability (expanding, diminishing, or stable) of the groundwater contaminant plume (Weidemier *et al*. 1999).

Table 3. Sampling frequency optimization based on contaminant concentration trends in monitoring wells.

CONCENTRATION TREND INDICATED BY MK TEST EVALUATION	INFERRED CONTAMINANT FLUX AND PLUME STABILITY	GROUNDWATER SAMPLING FREQUENCY
Increasing	Increased flux, expanding plume boundary	Maintain or Increase
Decreasing	Decreased flux, diminishing plume	Maintain or Reduce
Stable	Unchanged flux, stable plume	Reduce
No Trend	Not determined	Maintain

CONCLUSION

This paper describes the use of the MK Test in the evaluation of groundwater monitoring data to determine whether selected time-series data indicate statistically significant trends. Application of this approach has proven an effective and relatively non-subjective means to base a range of decisions regarding LTM of groundwater contamination, such as groundwater sampling frequency.

REFERENCES

Gilbert, R.O. 1987. *Statistical Method for Environmental Pollution Monitoring*, Chapter 16.4 Mann-Kendall Test, pp 208-217, and Appendix A, Table A.18 Probabilities for Mann-Kendall Non-Parametric Test for Trend, p. 272. John Wiley & Sons, Inc., New York, NY.

Hollander, M. and D. A. Wolfe. 1973. *Nonparametric Statistical Methods*, Table A.21. John Wiley & Sons, New York, NY.

Kendall, M.A. 1975. *Rank Correlation Methods*. Charles Griffin, London, UK.

Wiedemeier, T.H., C.J. Newell, H.S. Rifai, and J.T. Wilson. 1999. *Natural Attenuation of Fuels and Chlorinated Solvents in the Subsurface*, Chapter 7.3 Statistical Tests for Plume Stability, pg. 316-321. John Wiley & Sons, New York, NY.