### Probabilistic Modeling of Landfill Subsidence Introduced by Buried Structure Collapse – 13229

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## ABSTRACT

The long-term reliability of land disposal facility final cover systems – and therefore the overall waste containment – depends on the distortions imposed on these systems by differential settlement/subsidence. The evaluation of differential settlement is challenging because of the heterogeneity of the waste mass and buried structure placement. Deterministic approaches to long-term final cover settlement prediction are not able to capture the spatial variability in the waste mass and subgrade properties, especially discontinuous inclusions, which control differential settlement. An alternative is to use a probabilistic model to capture the non-uniform collapse of cover soils and buried structures and the subsequent effect of that collapse on the final cover system. Both techniques are applied to the problem of two side-by-side waste trenches with collapsible voids. The results show how this analytical technique can be used to collapse and the effective thickness of the cover soils. This approach allows designers to specify cover thickness, reinforcement, and slope to meet the demands imposed by the settlement of the underlying waste trenches.

## INTRODUCTION

Differential settlement threatens the functionality of final cover systems at important waste disposal facilities as documented at several sites [1][2][3][4]. The long-term reliability of land disposal facility final cover systems – and therefore the overall waste containment – depends on the distortions imposed on these systems by differential settlement/subsidence [5]. Differential settlement is caused by inconsistent compaction, void space distribution, collapsing buried structures, waste material stiffness, time-dependent primary compression of the fine-grained soil matrix, long-term creep settlement of the soil matrix and the debris, etc. The evaluation of differential settlement is challenging because of the heterogeneity of the waste mass and buried structure placement.

To properly analyze the effects of these factors on final cover performance, a number of analytical tools are required: 1) a model of the surface expression of subsurface void collapse, 2) a metric for final cover performance, and 3) a probabilistic model to capture the uncertain collapse pattern and spatial variability of subsurface features. Equations that address the first two needs can be developed deterministically (i.e., without consideration of probabilistic effects). To properly address subsurface uncertainties and apparently random spatial distributions of subsurface properties, a probabilistic model is needed.

Deterministic approaches to long-term final cover settlement prediction are not able to capture the spatial variability in the waste mass and subgrade properties, especially discontinuous inclusions, which control differential settlement. Deterministic approaches are particularly poorly suited to treat the problem of irregular collapse patterns of buried structures during their buried life. An alternative, probabilistic solution is to use a stochastic model to capture the non-uniform collapse of buried structures and the subsequent effect of that collapse on the final cover system.

In the following sections, a deterministic model of final cover subsidence is developed to treat this problem. An example series of two side-by-side waste trenches is analyzed to explore the effects of subsurface volume loss on final cover performance. Following the development of this model, a stochastic subsurface structure collapse model is applied to illustrate the impact of uncertain subsurface

conditions. The modeling effort informs the design, construction, operation, and maintenance of land disposal facilities. For example, this approach allows a rational assessment of buried structure placement strategies (horizontal spacing, vertical spacing, burial depth, backfill placement requirements, etc.). Also, this approach allows a performance-based assessment and specification of backfill compaction and reinforcement. This assessment is enabled by quantifying the direct impact of cover soil and trench geometry and properties on final cover performance via the calculated post-settlement percent inundated area. The procedures to develop this model and calculate the post-settlement percent inundated area are presented.

The presented methodology is useful for the planning and development of new land disposal facilities. It is designed to inform the process for specifying waste placement patterns, cover thickness, cover reinforcement, and in-place testing and inspection. This methodology can also be used to re-evaluate the long-term differential settlement behavior at existing closed land disposal facilities to identify problematic developments and inform possible remedial action (e.g., reinforcement of cover soils). This capability is crucial since it is impractical to construct experimental land disposal facilities to different specifications and monitor their performance over the required lifetime of these facilities. The ability to construct rational stochastic models of future performance is imperative.

The results of this study show that is practical to model this future performance within a rational, conservative, probabilistic framework. The tools developed in this study are readily applicable to the analysis of long-term land disposal facility final cover performance. Through trial waste configurations and subsequent performance predictions, the presented methodology allows a quantitative approach to waste placement specification.

# METHODOLOGY

## Waste Trench Design Scenario

Figure 1 illustrates the design scenario analyzed. Two side-by-side waste trenches backfilled with containers and soil fill are modeled. The dimensions of the trenches are given in Figure 1. Figure 2 illustrates the final cover configuration and highlights the overall design problem. The long-term collapse of subsurface voids (containers, bulked soil backfill, etc.) leads to the creation of surface depressions due to differential settlement. These surface depression affect the performance of the surface water management system, impairing flow and possibly causing ponding. The differential settlement causing these surface depressions is the direct result of an uneven distribution of subsurface compressibility and voids (i.e., trenches versus undisturbed soil, containers versus backfill, loose fill versus densified fill). The transmission of subsidence to the surface depends on the thickness, stiffness, and strength of the overlying cover soils. The following sections detail the modeling of this transmission and the effects of changes in the cover soil properties. The probabilistic part of the methodology addresses the non-uniform distribution of collapsible voids in the subsurface.

The primary purpose of the final cover is diverting surface water from the land disposal facility for collection and discharge, thereby preventing infiltration into the buried waste. The ability of the final cover to fulfill this purpose is impaired by differential settlement [4][5]. The percent area inundated,  $A_I$  (%) is proposed as a metric to quantify the performance of the final cover system with respect to differential settlement. This ratio represents the proportion of the final cover area where water will pond on top of the final cover and be unable to drain via gravity to surface water collection points. The inundated area is computed by filling depressions in the computed post-settlement final cover profile with a horizontal surface until the margins of the filled area reach the local maxima from where water drainage paths would reach the edge of the analyzed surface. The intersection of this horizontal surface with the

post-settlement final cover profile defines the edges of the inundated area. The ratio of the inundated area to the overall modeled final cover area defines the percent area inundated,  $A_I(\%)$ .



Figure 1. Illustration of waste placement trenches (not to scale).



Figure 2. Illustration of final cover system and differential settlement.

#### Deterministic volume loss model

Different equations are traditionally used to model surface subsidence resulting from subsurface volume loss due to tunneling [6]. These equations are useful for modeling and understanding the problem of subsidence above waste trenches because the two problems have many mechanical aspects in common (e.g., collapse of discrete voids at depth, subsidence distributed through overlying soils, etc.). One widely used model that has been demonstrated to accurately predict the shape of surface deformations due to subsurface volume loss is based on the normal (Gaussian) distribution. For tunnel modeling, it is typically applied in one dimension assuming plane strain conditions along the axis of the tunnel. For example, the 1-dimensional, plane strain distribution of vertical surface subsidence  $\Delta z$  perpendicular to the tunnel axis can be expressed

$$\Delta z = \frac{\Delta V}{x_i \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x}{x_i}\right)^2}$$
(Eq. 1)

where  $\Delta V$  is the volume loss at depth,  $x_i$  is the distance to the point of inflection in the surface depression (the standard deviation of the normal distribution), and x is the horizontal distance from the tunnel centerline. Parameter  $x_i$  depends on the thickness of the overlying soils and their resistance to deformation. The following equation is a commonly-applied relationship to determine  $x_i$  [6].

$$x_i = kH \tag{Eq. 2}$$

where empirical proportionality constant k depends on the soil properties. Constant k can range between 0.4 and 0.7 for cohesive soils and between 0.2 and 0.3 for granular soils. Constant k is assumed to be 0.3 for the present analysis. The results section will discuss how the soil profile can be engineered to affect this value. For the current problem, it is useful to consider the distribution of surface subsidence in two dimensions. Therefore, the bivariate normal distribution is used,

$$\Delta z = \frac{\Delta V}{2\pi x_i} e^{-\frac{1}{2} \left(\frac{x^2 + y^2}{x_i}\right)}$$
(Eq. 3)

where x and y are the horizontal distances from the center of the void. These distributions are defined so that

$$\iint_{-\infty}^{\infty} \Delta z dx dy = \Delta V \tag{Eq. 4}$$

therefore satisfying the condition that volume lost at depth is fully expressed at the ground surface.

To model the ponding depicted in Figure 2, a surface defining the design final cover elevations is first established. Next, a 1 m by 1 m grid of volume loss areas is established corresponding to the geometry of the trenches depicted in Figure 1. In the deterministic analysis, the volume loss  $\Delta V$  assigned to each of these 1 m<sup>2</sup> areas is set to a fixed percentage of the total trench volume under that area. Soil cover thickness H is calculated as the vertical distance from the top of the trench to the top of the final cover. This value depends on both the design minimum cover thickness and the design final cover slope. Due to the slope of the final cover, thickness H varies over the analysis domain. Once these parameters are assigned, Equations 2 and 3 are then used to calculate the settlement at each point on the final cover surface. This procedure requires the calculation of settlement at each point of the final cover due to the subsidence caused by each of the discrete trench areas defined a value of  $\Delta V$ . Accordingly, Equations 2 and 3 are repeated thousands of times to complete the analysis. Therefore, the analysis is conducted within a programmable spreadsheet and the solutions are presented graphically in the results section. Following the calculation of the post-settlement final cover profile, the inundated area and the corresponding percent inundated area  $A_1(\%)$  are calculated to evaluate the post-settlement performance of the final cover.

#### **Random Field Model**

All soils and waste materials derive their physical properties from their constituent materials, their method of deposition, and their stress history. The complexity of the processes involved in a specific waste's or soil's placement and its constituent properties make complete characterization practically impossible. Furthermore, the subsidence of these features is expected to proceed non-uniformly due to these variations. Accordingly, engineers can measure only a few of the limitless variables governing compressible subgrade properties. These few variables can explain some, but not all of the variation observed in soil and waste, creating a scattered or random appearance to their behavior. This apparent randomness results from the unknown variables. Using statistics, the degree of this randomness can be described mathematically [7].

Using probabilistic models to describe the variability of a property measurement at one point in space is commonplace and introduced to engineers in school. Probabilistic models to simulate a continuum of values in space are known as random fields and are not in common use. Random field modeling can be understood as the random assignment of property values to different points in space. To realistically model a real physical system, certain rules governing this random assignment must be applied. The two most important rules applicable to the current problem are property variation and spatial correlation.

Spatial correlation describes the relationship between values sampled some distance apart. Values close to one another are more likely to be related. Conversely, values far apart are expected to be unrelated. Physically, soils and wastes placed in the same location are expected to have similar source materials, deposition, and stress histories compared to samples taken from different locations. This concept is expressed mathematically using a correlation function, which can be used to govern the selection of random values in a simulated random field. Therefore, a correlation function is needed to simulate a real physical system.

Because the exact distribution of the eventual volume loss  $\Delta V$  values throughout the trenches is not known in advance, it is necessary to simulate possible distributions to model likely final cover performance. Random fields are generated using Local Average Subdivision [8][9]. In the present analysis, the Local Average subdivision is used to model the spatial distribution of  $\Delta V$  rather than soil properties. Volume loss  $\Delta V$  depends on a number of material-specific phenomena that are not discussed in this paper.

In this analysis, random values of  $\Delta V$  are generated according to a normal distribution that observes a spatial correlation rule. Similarly to previous studies of differential settlement using random fields [10], the spatial correlation is modeled using the following correlation function:

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right)$$
(Eq. 5)

where  $\rho$  is the correlation coefficient,  $|\tau|$  is the absolute distance between two points being modeled ("the lag distance"), and  $\theta$  is the scale of fluctuation. Scale of fluctuation  $\theta$  can be understood as the distance at which field values are no longer significantly correlated. For this analysis a value of  $\theta = 1$  m is applied.

A single instance of a simulated random field is known as a realization. Random field modeling typically involves generating many realizations to assess the likelihood of various calculated outcomes. In the following analysis results, two realizations are used to illustrate this procedure. Similarly to the deterministic analyses, the random field generated values of  $\Delta V$  are used to calculate the post-settlement elevations of the final cover system using Equations 2 and 3. These elevations are then used to calculate the percent area inundated A<sub>I</sub>(%).

### **RESULTS AND DISCUSSION**

#### **Deterministic Model**

Figure 3 presents the results of a deterministic analysis of the settlement of a 1% slope final cover with a minimum cover thickness of 1.5 m. For this analysis, a volume loss  $\Delta V$  corresponding to 1% of the trench volume was applied uniformly over the footprint of the trenches. Despite this modest volume loss, with this final cover thickness, the impact of the individual trenches on final cover distortion is readily apparent, agreeing with field observations [4]. In the specific instance of a 1% design final cover slope, the result is a significant percent inundated area (indicated in yellow in Figure 3). The analysis shown in Figure 3 indicates a 27.8% inundated area for this scenario.



**Figure 3.** Deterministic differential settlement analysis results for 1% design slope, 1.5 m soil cover, 1% volume loss case.

The deterministic analyses were repeated for a number of different minimum final cover depths and design final cover slopes to illustrate the application of these analyses to a design problem. Figures 4 through 7 illustrate a few example results from these calculations. Figure 4 (showing 11.6% inundated area), when compared to Figure 3 (showing 27.8% inundated area), illustrates the effect of increasing the final cover design slope on final cover performance. As the design slope increases, the inundated area decreases, allowing more of the final cover to drain despite differential settlement. Figure 5 (showing 4.25% inundated area) illustrates the effect of increasing the minimum soil cover. As the soil cover thickness increases, the volume loss is distributed over a wider area, resulting in more gradual changes in slope. Accordingly, more of the final cover area drains in Figure 5 than in Figure 4. Also, due to the overlapping zone of influence between the two trenches, the lower portion of the final cover ponds while the upper portion does not. Figure 6 (showing 12.4% inundated area) emphasizes this point, showing the effect of a thicker cover on the 1% design slope case. Finally, Figure 7 (showing 31.1% inundated area) shows the effects of increasing the volume loss to 2% for a 1% slope over a 9 m minimum final cover.



**Figure 4.** Deterministic differential settlement analysis results for 2% design slope, 1.5 m soil cover, 1% volume loss case.

When designing or retrofitting a final cover system, these analyses are used to assess whether the proposed design will achieve a satisfactory level of performance, quantified by the percent inundated area. As expressed in Equation 2, the type of soil, hence its properties, influence the surface distribution of volume loss. Soil type in this instance is a proxy for its mechanical properties – strength and stiffness. Therefore, the soil stiffness and strength will alter the distribution of settlement. Denser, stiffer soils will spread deformation out over a broader area, blunting the effects of differential settlement. Also, geosynthetic reinforcement can be added to the cover soil, increasing its stiffness and strength [11]. In terms of these analyses, the introduction of reinforcement increases the effective depth of cover soil.



Figure 5. Deterministic differential settlement analysis results for 2% design slope, 3 m soil cover, 1% volume loss case.

The effect of increasing the effective thickness of the cover soil, either through additional soil, soil improvement, or soil reinforcement, is to decrease the percent area inundated. Figures 8 and 9 summarize the results of analyses of several different combinations of design cover slope, minimum cover thickness, and volume loss. These figures show that surface water management improves as the design slope is increased and as the effective cover thickness is increased. Figure 8 presents the results for 1% volume loss cases while Figure 9 presents the results for 2% volume loss cases. The difference between Figure 9 and 8 can be seen in the comparatively large inundated area values in Figure 9. These charts can be used as design charts to select an optimal combination of design slope and cover thickness to meet the project design performance targets. Specific charts conforming to site-specific trench configurations and waste filling practices can be developed using these procedures.



Figure 6. Deterministic differential settlement analysis results for 1% design slope, 6 m soil cover, 1% volume loss case.



Figure 7. Deterministic differential settlement analysis results for 1% design slope, 9 m soil cover, 2% volume loss case.



Figure 8. Deterministic differential settlement analysis results for 1% volume loss.



Figure 9. Deterministic differential settlement analysis results for 2% volume loss.

### **Probabilistic Model**

Figures 10 and 11 present two replicate realizations of a probabilistic analysis of the 3% slope, 6 m minimum cover, 2% mean  $\Delta V$  design case. Compared to Figures 3 through 7, the inundated area calculated for Figures 10 and 11 is irregularly-shaped, reflecting expected behavior in the field. The causative irregularity in the settlement profile is the direct result of the random field-modeled heterogeneous subsidence profile for the trenches. In the case of Figures 10 and 11, a coefficient of variation (COV) = 30% was applied to the 2% mean value of  $\Delta V$ . This COV value is typical for soil compressibility [7] and is therefore representative of soil backfill materials. As mentioned in the methodology,  $\theta = 1$  m was used to generate the random field for both Figures.

Also noteworthy in Figures 10 and 11 is the variation in  $A_I$  (%) (4.06% versus 4.86%) between the two realizations. This result is expected since each realization is a unique simulation of a possible outcome of the design. In order to quantify the overall probabilistic performance of a final cover design, several more realizations would be generated and statistics performed on the resulting values of  $A_I$  (%) to characterize the confidence in the design performance target. If the occurrence of unsatisfactory values of  $A_I$  (%) is too high, then the design must be altered to improve final cover performance. If the design goal is to achieve  $A_I$  (%) = 0% for a significant number of simulations (e.g., 95%), then several simulations of marginal designs (designs where inundation is slightly prevented) will be needed. Since deterministic analyses can only return a single estimate of  $A_I$  (%), they are misleading for these marginal design cases. As illustrated by the difference in  $A_I$  (%) computed for Figures 8 and 9, deterministic analyses can predict zero occurrence of  $A_I$  (%) > 0 when, in reality, this outcome would have non-zero likelihood. Only probabilistic analyses can quantify the likelihood of poor performance for these marginal designs.



Figure 10. Probabilistic differential settlement analysis results for 3% design slope, 6 m soil cover, 2% volume loss case  $-1^{st}$  realization.



Figure 11. Probabilistic differential settlement analysis results for 3% design slope, 6 m soil cover, 2% volume loss case  $-2^{nd}$  realization.

### CONCLUSIONS

Differential settlement and its adverse impacts on final cover performance are an established and ongoing challenge to landfill designers. An analysis technique to model the impacts of waste and cover soil volume loss on final cover subsidence has been presented. A specific design scenario incorporating two side-by-side disposal trenches was presented to illustrate the use and results of the model. This analysis is shown to capture many of the relevant features of the final cover settlement problem that have been observed in the field. The analyses show significant impacts on typical cover designs, even from relatively modest volume losses in the trenches. Therefore, soil improvement (e.g., dynamic compaction, grouting), such as geosynthetic reinforcement, is indicated for similar designs. The effects of this improvement can be modeled by increasing the effective cover thickness resulting from cover soil modification. Increases in effective cover thickness result in a reduction in the inundated area.

The overall performance of final cover systems subject to differential settlement has been quantified using the percent area inundated. This simple model result allows a quantitative comparison of alternative designs. In order to optimize final cover slope and thickness, the relative risk of unacceptable inundated area values must be computed. This risk can only be computed using probabilistic methods. A probabilistic approach is especially valuable for marginal design cases where deterministic analyses will erroneously indicate zero occurrence of ponding for final covers. Because most engineered systems are designed to operate at some specified performance level relative to the marginal design case, these analyses are needed to rationally design waste disposal facility final covers.

Therefore, the suggested design methodology emerging from this approach is to 1) perform a rough design optimization using the deterministic analyses, evaluating the trade-off between design slope and cover thickness and 2) perform a probabilistic analysis to refine the design and to quantify the confidence level in the design's performance. For new land disposal facilities, by quantifying the design risk, the design engineers can communicate with other project stakeholders regarding the projected design performance, design optimization criteria and decisions, and uncertainties. For existing facilities, this approach allows engineers to model existing behavior and evaluate the effects of proposed improvements on future performance.

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