# Pumping Test Characterization of Deep Vadose Zone Properties - 11415

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## ABSTRACT

To analyze contaminant transport in a deep vadose zone (DVZ) such as that found beneath the 200 Area of Hanford one must characterize its saturated and unsaturated hydraulic properties. There presently are no known ways to do so directly on the basis of hydraulic tests other than those conducted in the laboratory on small, disturbed and widely spaced samples of sediment extracted from boreholes. We propose a method of characterizing DVZ hydraulic properties on realistic field scales by means of pumping tests conducted in the underlying saturated zone. The proposed method allows inferring saturated and unsaturated hydraulic properties of both the DVZ and the underlying pumped zone from groundwater drawdown and recovery data recorded in the saturated and, preferably but optionally, in the unsaturated zone above the water table. We describe briefly the theoretical basis for the proposed method; investigate the effects of unsaturated zone constitutive parameters and thickness on drawdowns in the saturated and unsaturated zones as functions of position and time; demonstrate the development of significant horizontal hydraulic gradients within the DVZ in response to pumping; validate our theoretical analysis against numerical simulations of drawdown in a synthetic aquifer having unsaturated properties described by a widely used constitutive model; and illustrate it in the context of a real pumping test.

## INTRODUCTION

Throughout the DOE complex, and particularly at Hanford, contaminants have migrated from shallow disposal facilities to groundwater at depth via the vadose zone. Neither the precise manner or rates of this migration nor the fate or spatial distribution of contaminants throughout the vadose zone are known with certainty. Contaminants presently in the vadose zone pose a risk of continued leakage to groundwater. The DOE faces a daunting challenge insuring that contaminants in the near surface and within the vadose zone do not pose unacceptable future risks to humans and the environment. The agency must assess quantitatively such risks under a range of potential site management options in a way deemed credible by regulators and the public. Risk quantification entails the use of models simulating flow, transport and the fate of contaminants vertically and laterally through the vadose zone. A particular challenge is posed by deep portions of this zone which are presently not amenable to field-scale characterization; the only current option is to rely on laboratory analyses of relatively small and partly disturbed samples of sediment extracted from a few sparsely spaced boreholes. This paper describes a novel methodology, and underlying theory, that should allow the DOE to infer field-scale hydraulic properties of the saturated zone (horizontal and vertical saturated hydraulic conductivity, compressive specific storage and specific yield or drainable porosity) and unsaturated properties of deep vadose zone material above the water table (parameters of constitutive functional relationships between volumetric water content or saturation, relative hydraulic conductivity and capillary pressure head) on the basis of pressure variations recorded in wells and piezometers completed within the saturated zone, and optionally variations in pressure and/or saturation recorded in the unsaturated zone, when water is withdrawn at controlled rate from a well fully or partially penetrating the saturated zone.

## THEORY

We consider a compressible unconfined aquifer of large lateral extent resting on an impermeable boundary (Figure 1). For simplicity the aquifer is taken to be spatially uniform and anisotropic, with constant specific storage $S_s$ and fixed ratio $K_D = K_z / K_r$ between vertical and horizontal saturated hydraulic conductivities, $K_z$ and $K_r$, respectively. The aquifer is saturated beneath an initially horizontal water table at elevation $z = b$ defined as a $\psi = \psi_a$ isobar where $\psi$ is pressure head and $\psi_a \le 0$ is the pressure head required for air to enter a saturated medium. A saturated capillary fringe at non-positive pressure $\psi_a \le \psi \le 0$ extends from the water table down to the $\psi = 0$ isobar (corresponding to the traditional definition of a water table) at elevation $b + \psi_a$ (note that the capillary fringe disappears if one sets $\psi_a = 0$). Prior to the onset of pumping the saturated and overlying unsaturated zones are at uniform initial hydraulic head $h_0 = b + \psi_a$. Starting at time $t = 0$, water is withdrawn at a constant volumetric rate $Q$ from a well of finite radius $r_w$ and wellbore storage coefficient $C_w$ (volume of water released from storage in the pumping well per unit drawdown in this well). The pumping well penetrates the saturated zone between depths $l$ and $d$ below the initial water table (at air entry pressure).

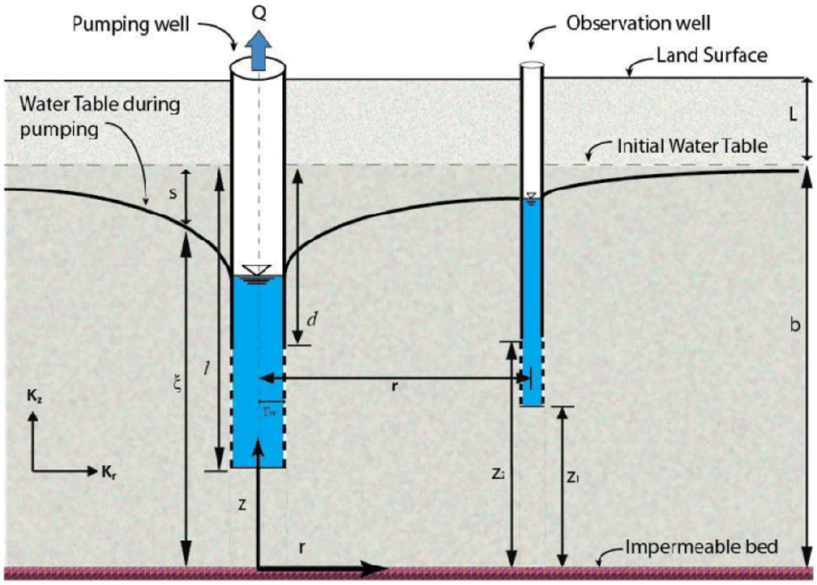

Figure 1. Schematic representation of system geometry.

We represent the water retention properties of unsaturated aquifer materials above the water table (i.e., the deep vadose zone) by means of an exponential function

$$S_e = \frac{\theta(\psi) - \theta_r}{S_y} = e^{a_c(\psi - \psi_a)} \qquad a_c \ge 0 \qquad \text{(Eq. 1)}$$

where $S_e$ is effective saturation, $\theta_r$ is residual water content and $S_y = \theta_s - \theta_r$ is drainable porosity or specific yield. The relative hydraulic conductivity of this material is represented in a similar manner by

$$k(\psi) = \begin{cases} e^{a_k(\psi - \psi_k)} & \psi \le \psi_k \\ 1 & \psi < \psi_k \end{cases} \qquad a_k \ge 0, \qquad \text{(Eq. 2)}$$

with parameters $a_k$ and $\psi_k$ that generally differ from $a_c$ and $\psi_a$ in (Eq. 1). The parameter $\psi_k \le 0$ represents a pressure head above which relative hydraulic conductivity is effectively equal to unity, which is sometimes but not always equal to the air entry pressure head $\psi_a$. In addition to being mathematically tractable, these exponential expressions are sufficiently flexible to provide acceptable (though not perfect) fits to standard constitutive models such as those mentioned earlier.

[1] provides mathematical expressions that allow evaluating dimensionless point drawdowns (at given radial distances $r$ from the axis of the pumping well, elevations $z$ above the bottom of the saturated zone and times $t$ following the onset of pumping) $s_D(r_D, z_D, t_s) = (4\pi K_r b / Q) s(r_D, z_D, t_s)$ in the saturated zone and $\sigma_D(r_D, z_D, t_s) = (4\pi K_r b / Q)\sigma(r_D, z_D, t_s)$ in the overlying unsaturated zone where $s(r_D, z_D, t_s)$ and $\sigma(r_D, z_D, t_s)$ are actual drawdowns (drops in hydraulic head in response to pumping) in these two zones, respectively, $r_D = r/b$ is dimensionless (normalized by initial saturated zone thickness) radial distance, $z_D = z/b$ is dimensionless elevation and $t_s = K_r t / S_s r^2$ is dimensionless time. Drawdown in an observation well that penetrates the saturated zone

between elevations $z_{D1} = z_1 / b$ and $z_{D2} = z_2 / b$ (Figure 1) is obtained by averaging the point drawdown over this interval according to

$$s_{z_{D2}-z_{D1}}(r_D, t_s) = \frac{1}{z_{D2} - z_{D1}} \int_{z_{D1}}^{z_{D2}} s(r_D, z_D, t_s)\, dz_D . \qquad \text{(Eq. 3)}$$

When water level is measured in a piezometer or observation well having storage coefficient $C$ (volume of water stored in the instrument per unit increase in hydraulic head) the response of the measuring instrument is delayed in time. In a manner analogous to [2 - 3] we write

$$s = s_m + t_B \frac{\partial s_m}{dt} \qquad \text{(Eq. 4)}$$

where $s$ is actual drawdown, $s_m$ is measured (delayed) drawdown, $t_B = C / FK$ is basic (characteristic) time lag of the instrument, $F$ is a shape factor (having dimensions of length) and $K$ is an equivalent isotropic hydraulic conductivity of the formation. The characteristic time lag $t_B$ can be determined by means of a slug or pulse test, obviating the need to know $C$, $F$ or $K$. Integrating (Eq. 4) allows expressing $s_m$ in terms of $s$ via

$$s_m = s\left[1 - e^{-t/t_B}\right] \qquad \text{(Eq. 5)}$$

or its dimensionless equivalent [3]

$$s_{mD} = s_D\left[1 - e^{-t_s/t_{Bs}}\right] \qquad \text{(Eq. 6)}$$

where $t_{Bs} = K_r t_B / S_s r^2$.

## NUMERICAL VERIFICATION OF THEORY

To verify our theory we compare it with numerical simulations using the STOMP code [4]. In particular, we simulate flow in an isotropic aquifer $(K_D = 1.0)$ with horizontal hydraulic conductivity $K_r = 1.0 \times 10^{-4}\ m/s$, specific storage $S_s = 4.0 \times 10^{-5}\ m^{-1}$ and specific yield $S_y = 0.322$. In the numerical model, water retention and relative hydraulic conductivity are described by the widely used constitutive model of van Genuchten [5] and Mualem [6] with parameters $\log \alpha = -1.453$, $\theta_s = 0.375$, $\theta_r = 0.053$ and $\log n = 0.502$ typical of sandy soils. In our analytical solution we set $\psi_a = 14.2$ cm, $\psi_k = 8.3$ cm, $a_k = 8.2$ m$^{-1}$ and $a_c = 3.4$ m$^{-1}$, values estimated by [7] upon fitting (1) and (2) by least squares to the van Genuchten - Mualem model with the above parameters. The saturated-unsaturated aquifer rests on an impermeable horizontal boundary, has a combined thickness of 9 m and a water table (defined here as a zero-pressure isobar) located initially 2.75 m below its horizontal land surface boundary. The pumping well has an inner diameter of 0.13 m, penetrates the bottom 3.65 m of the aquifer and discharges at a rate of 40 l/min. We simulate it as a highly permeable medium with specific storage 100 times larger than that of the aquifer, yielding a dimensionless wellbore storage coefficient of $C_{wD} = C_w / \pi S_s b r_w^{\ 2} = 58.2$ where $C_w$ is the actual storage coefficient (volume of water released from the well per unit drop in head) of the pumping well.

Figure 2 compares time-drawdowns predicted analytically and simulated numerically at three radial distances $r$ = 0.26, 0.80 and 1.51 m from the axis of the pumping well at elevation $z = 2.0$ m above the aquifer bottom in the saturated zone. The agreement is acceptable at all three points, at all times, being least satisfactory at intermediate times but tending to improve as radial distance and time increase.

Figure 3 compares time-drawdowns at radial distance $r = 1.51\, m$ and elevation $z = 2.0\, m$ in the saturated zone obtained (1) analytically with our solution and numerically with STOMP upon considering pumping well storage but ignoring delayed piezometer response or, more specifically, upon setting $C_{wD} = 58.2$ and $t_b$ = 0 min, and (2) analytically upon setting $C_{wD} = 0$ and $t_b$ = 0.011 min. Figure 3 makes clear that the two cases are difficult to distinguish from each other, i.e., that it may be hard to differentiate between the effects of pumping well and piezometer storage based solely on observed drawdowns. Correspondingly, it may be difficult to estimate the corresponding storage parameters simultaneously on the basis of drawdown data alone.

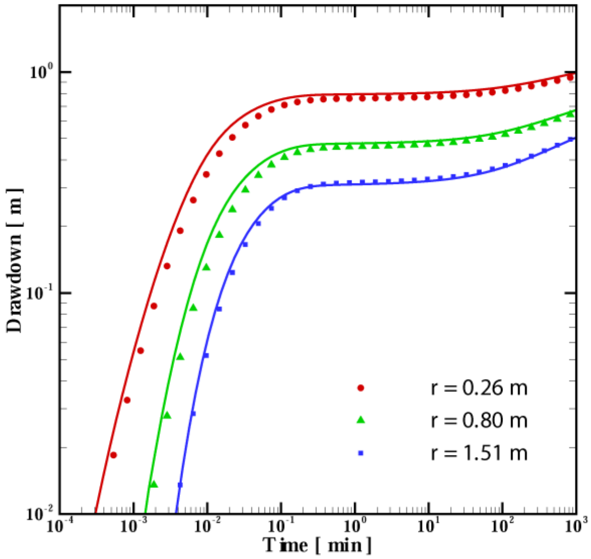

Figure 2. Drawdowns as functions of time computed analytically (solid) and numerically with STOMP (symbols) for synthetic pumping test at $r$ = 0.26, 0.80 and 1.51 m and $z = 2.0$ m in saturated zone.

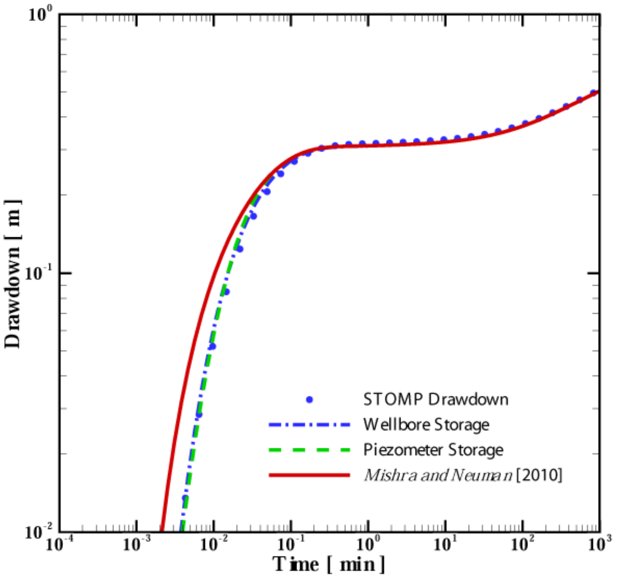

Figure 3. Time-drawdowns at radial distance $r = 1.51$ m and elevation $z = 2.0$ m in saturated zone obtained (1) analytically with our solution and numerically with STOMP upon setting $C_{wD} = 58.2$ and $t_B = 0$ min, and (2) analytically upon setting $C_{wD} = 0$ and $t_B = 0.011$ min.

Figure 4 compares time-drawdowns predicted analytically and simulated numerically at three radial distances $r$ = 1.51, 5.04 and 10.71 m from the axis of the pumping well at elevation $z = 6.61$ m in the unsaturated zone. The agreement is less good than in the saturated zone but acceptable at all three points, tending to improve as radial distance and time increase.

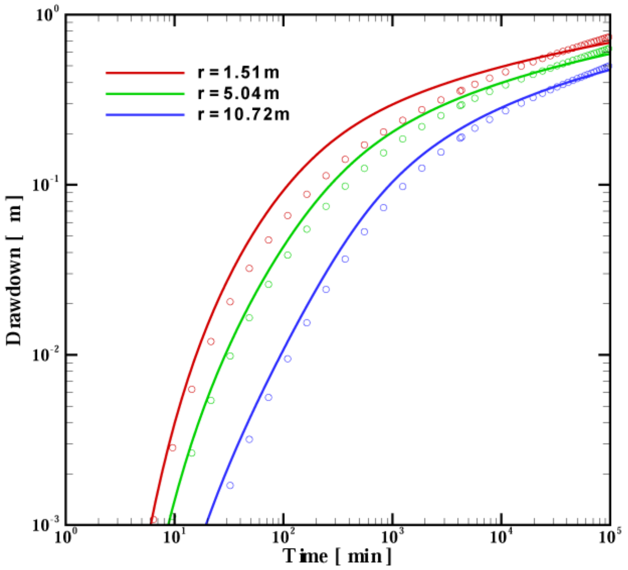

Figure 4. Drawdowns as functions of time computed analytically (solid) and numerically with STOMP (symbols) for synthetic pumping test at $r$ = 1.51, 5.04 and 10.71 m and $z = 6.61$ m in unsaturated zone.

## ANALYSIS OF AQUIFER TEST AT BORDEN, ONTARIO, CANADA

A 7-day pumping test was conducted during August 2001 by University of Waterloo researchers at the Canadian Forces Base Borden in Ontario, Canada. Details of the test are described in [8]. At the test site the aquifer is approximately 9.0 m thick, consists largely of unconsolidated medium-grain sand of glacio-deltaic or glacio-fluvial origin and overlies a clayey silt aquitard of relatively low permeability. A well screened between depths 2.6 and 6.25 m below the initial water table, located 2.75 m below the land surface, was pumped at a constant rate of 40 l/min for 7 days. Hydraulic head variations were monitored in 25 piezometers installed at diverse locations, distances and depths around the pumping well. Head variations in 11 piezometers and in the pumping well were measured with pressure transducers connected to data loggers. These 11 records, spanning a time interval from 2 seconds to 10,457 minutes, were fitted by us simultaneously to our analytical solution by least squares using the parameter estimation code PEST [9]. Similar to [10], we specified the characteristic time lag $t_B$ of each piezometer a priori by setting $C = \pi r_p^2$, $K = K_r$ and $F = 2\pi L_p / \left[ L_p / 2r_r + \sqrt{1 + \left( L_p / 2r_r \right)^2} \right]$ where $L_p$ and $r_p$ are the screen length and radius, respectively, of any piezometer [2]. The results of the fit are illustrated in Figure 5 and listed, together with corresponding 95% confidence intervals, in Table 1. Table 2 compares our parameter estimates with those obtained by Endres et.al. [11] on the basis of 11 transducer-measured drawdown records and by Moench [10] on the basis of drawdowns recorded in all 25 piezometers. Whereas our estimates of hydraulic conductivity and specific storage are similar to those obtained in [10 - 11], our estimate 0.301 of specific yield is somewhat higher than the value 0.284 in [11] but significantly higher than the value 0.25 arbitrarily assigned to the aquifer in [10]. Our estimate of specific yield is virtually identical with the value 0.30 reported by Nwankwor et al. [12] on the basis of laboratory drainage experiments on samples of aquifer material from the site.

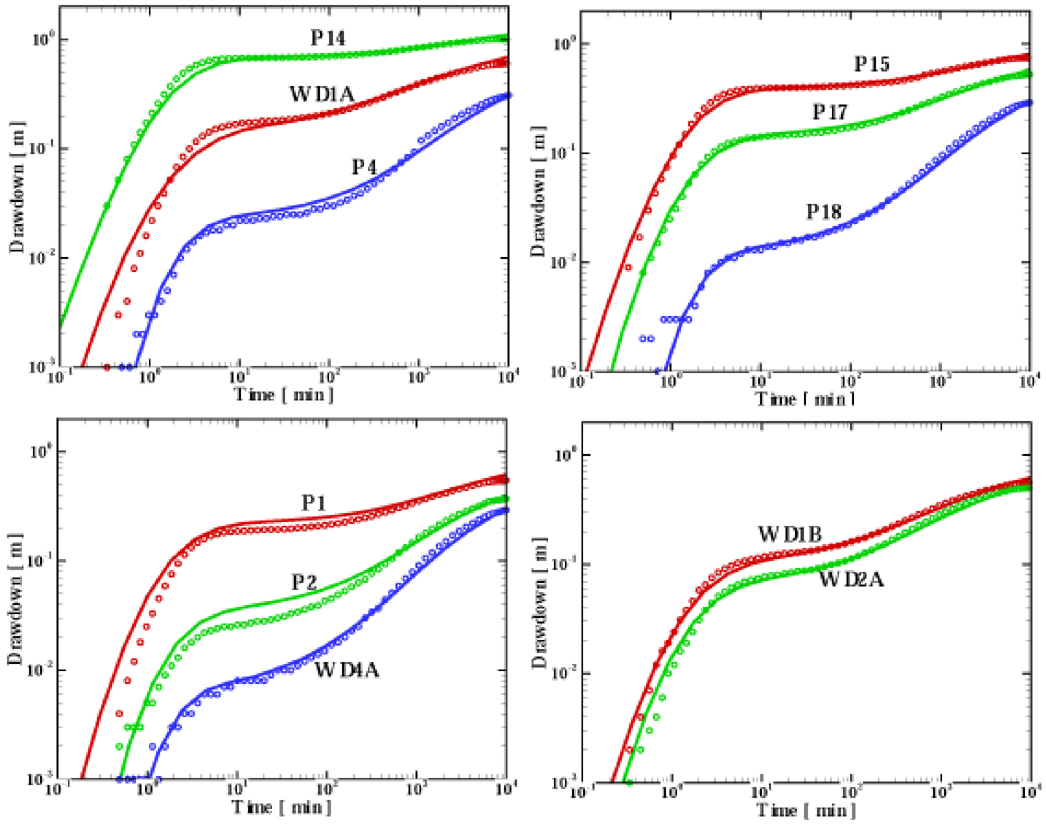

Figure 5. Simultaneous least squares fit of analytical solution (solid) to 11 water level records (symbols) at Borden measured by means of transducers.

Our estimates of exponential constitutive model parameters $a_c = 5.68\ m^{-1}$, $a_k = 23.66\ m^{-1}$ and $\psi_a - \psi_k = 3.12\ cm$ in Table 2 provide a least squares fit to van Genuchten [5] – Mualem [6] model parameters $\psi_a = 34.1\ cm$, $\psi_k = 30.98\ cm$, $\alpha = 2.32\ m^{-1}$ and $n = 5.85$. The corresponding exponential and van Genuchten - Mualem functions are plotted in Figure 6. Our pumping test estimate $\alpha = 2.32\ m^{-1}$ is somewhat larger and our $n = 5.85$ smaller than values $\alpha = 1.9\ m^{-1}$ and $n = 6.095$ obtained by Akindunni and Gillham [13] on the basis of laboratory drainage data from the site. Effective saturation values based on measured water contents above the water table in neutron access tube MBN − 5, located at a radial distance of about 5 m from the center of the pumping well, are seen in Figure 6 to lie very close to our pumping test derived effective saturation curve (recall that, under static initial conditions, capillary pressure head is equivalent to elevation above the zero pressure isobar).

Table 1. Parameter estimates and their 95 % confidence limits obtained by fitting our solution to 11 transducer-measured drawdown records from Borden piezometers shown by open circles in Figure 5.

| **Parameters** | **Estimated Value** | **95 % Confidence Limits** | |
|---|---|---|---|
| | | **Lower** | **Upper** |
| $K_r \ (m/s)$ | $6.36 \times 10^{-5}$ | $3.91 \times 10^{-5}$ | $7.13 \times 10^{-5}$ |
| $K_z / K_r$ | 0.45 | 0.37 | 0.56 |
| $S_s \ (m^{-1})$ | $5.67 \times 10^{-5}$ | $7.63 \times 10^{-6}$ | $2.44 \times 10^{-4}$ |
| $S_y$ | 0.301 | 0.223 | 0.389 |
| $a_c \ (m^{-1})$ | 5.68 | 3.81 | 12.61 |
| $a_k \ (m^{-1})$ | 23.66 | 6.32 | 44.31 |
| $\psi_a - \psi_k \ (cm)$ | 3.12 | 0.02 | 27.91 |
| $\log_{10}(C_{wD})$ | 3.44 | 2.83 | 4.12 |

Table 2: Comparison between parameter estimates obtained in this study on the basis of 11 transducer-measured drawdown records at Borden, those obtained based on the same records [11] and on all measured drawdowns [10].

| **Parameters** | **[11]** | **[10]** | **This study** |
|---|---|---|---|
| $K_r \ (m/s)$ | 6.10 × 10−5 | $6.84 \times 10^{-5}$ | 6.37 × 10−5 |
| $K_z \ (m/s)$ | 3.24 × 10−5 | 2.90 × 10−5 | 2.84 × 10−5 |
| $S_s \ (m^{-1})$ | 7.19 × 10−5 | 3.76 ×10−5 | 5.67 × 10−5 |
| $S_y$ | 0.284 | 0.25* | 0.301 |
| $a_c \ (m^{-1})$ | - | 5 | 5.68 |
| $a_k \ (m^{-1})$ | - | 31.7 | 23.66 |
| $\psi_a - \psi_k \ (cm)$ | - | - | 3.12 |

*Assigned (not estimated)

Finally we compare in Figure 7 drawdowns measured manually in 14 piezometers (shown by solid rectangles in Figure 11) with those predicted by our analytical solution with parameter estimates in Table 2 (based on transducer measurements in 11 other piezometers). The fit in all piezometers is fairly good. The fits in piezometers P13 and P16, located near the bottom of the aquifer (6.1 m below the initial water table), are less good due likely to the non-ideal nature of this boundary.

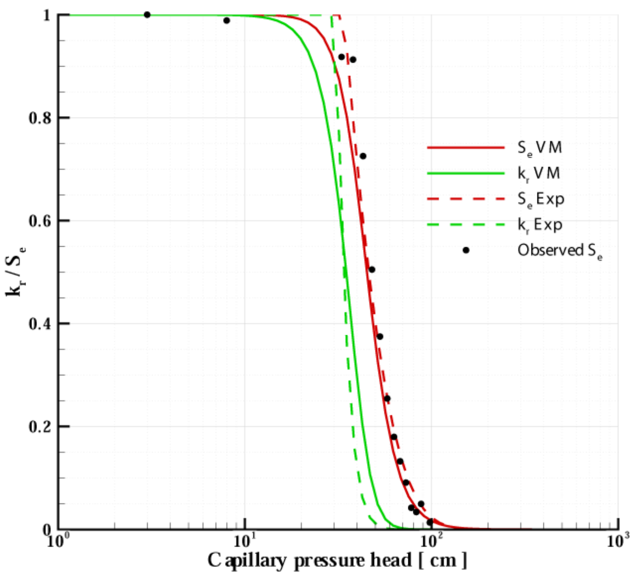

Figure 6. Least squares fit between exponential constitutive models with parameters $a_c = 5.68\ m^{-1}$, $a_k = 23.66\ m^{-1}$ and $\psi_a - \psi_k = 3.12\ cm$ and van Genuchten [5] - Mualem [6] model with parameters $\psi_a = 34.1\ cm$, $\psi_k = 30.98\ cm$, $\alpha = 2.32\ m^{-1}$ and $n = 5.85$. Also shown are effective saturations based on measured initial static water content profile above water table in neutron access tube MBN − 5.

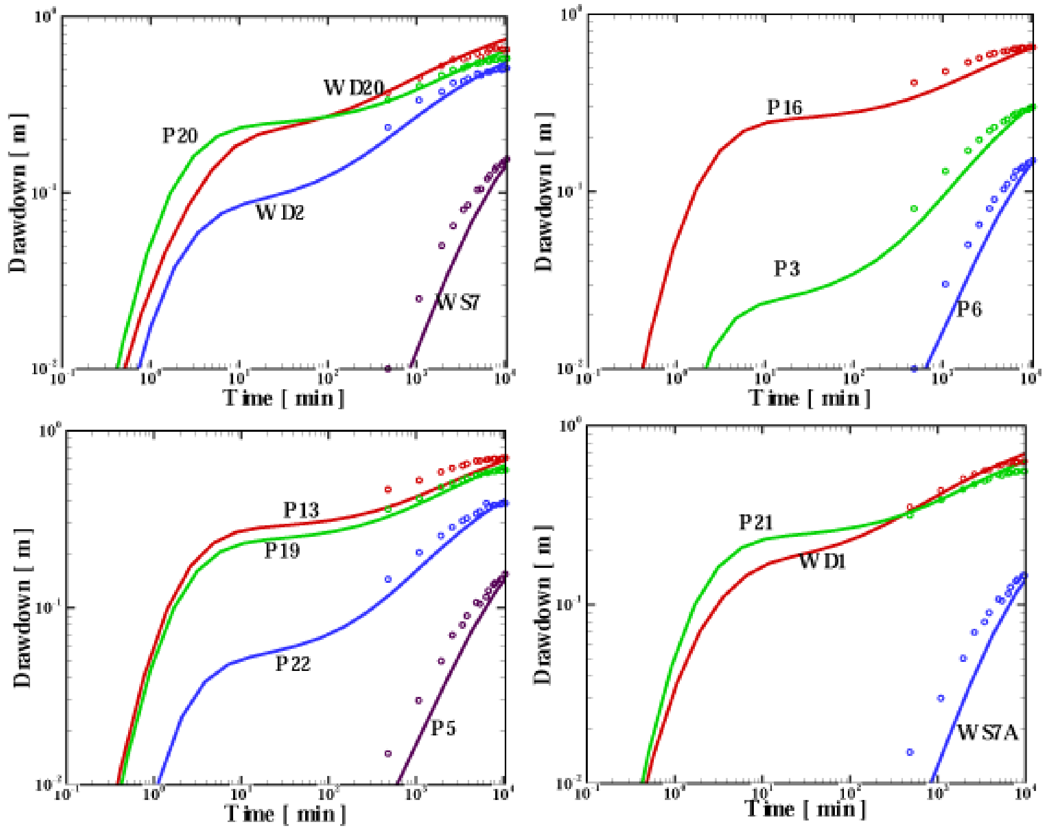

Figure 7. Predicted (solid) versus hand-measured (symbols) drawdowns in 14 Borden piezometers not used in parameter estimation.

## CONCLUSIONS

Our work leads to following major conclusions:

1. A novel method was proposed to characterize hydraulic properties of the deep vadose zone (DVZ) at sites such as Hanford on realistic field scales by means of pumping tests conducted in the underlying saturated zone. The method is based on a new analytical solution for axially symmetric saturated-unsaturated flow to a well with storage that partially penetrates the saturated zone of a compressible vertically-anisotropic unconfined aquifer. The solution allows considering delayed response of piezometers and observation wells due to water storage in these measuring devices.
1. The proposed method allows inferring saturated and unsaturated hydraulic properties of both the DVZ and the underlying pumped zone from groundwater drawdown and recovery data recorded in the saturated and, preferably but optionally, in the unsaturated zone above the water table.
2. The new analytical solution agrees with numerical simulations of drawdown in both the saturated and the unsaturated zones of a synthetic aquifer having unsaturated properties described by the van Genuchten [5] - Mualem [6] constitutive model. Agreement between the two solutions in the saturated zone is closer than in the unsaturated zone, least satisfactory at intermediate times but tending to improve as radial distance and time increase.
3. The analytical solution demonstrates (and our numerical simulations confirm) that it may be difficult to differentiate between pumping well and piezometer or observation well storage on the basis of drawdown data alone.
4. We used our solution to analyze 11 transducer-measured drawdown records from a seven-day pumping test conducted by University of Waterloo researchers at the Canadian Forces Base Borden in Ontario, Canada, and to validate our parameter estimates against manually-measured drawdown records in 14 other piezometers at the site. The validation was satisfactory except in two piezometers near the aquifer bottom due likely to the non-ideal nature of this boundary.
5. Similar to [10], we specified the characteristic time lag of each Borden piezometer a priori based on its radius and screen length. Our estimates of hydraulic conductivity and specific storage are similar to those obtained on the basis of 11 transducer-measured records in [11] and all 25 drawdown records in [10]. Our estimate 0.301 of specific yield is somewhat higher than the 0.284 value obtained in [11] but significantly higher than the 0.25 value assigned to the aquifer in [10]. Our estimate of specific yield is virtually identical to the 0.30 value reported in [12] on the basis of laboratory drainage experiments on samples of aquifer material from the site.
6. Estimates of exponential constitutive model parameters entering into our analytical solution provide a least squares fit to the van Genuchten [5] – Mualem [6] model with parameters $\psi_a = 34.1\ cm$, $\psi_k = 30.98\ cm$, $\alpha = 2.32\ m^{-1}$ and $n = 5.85$. Our pumping test estimate $\alpha = 2.32\ m^{-1}$ is somewhat larger and our $n = 5.85$ smaller than values $\alpha = 1.9\ m^{-1}$ and $n = 6.095$ obtained in [13] on the basis of laboratory drainage data from the site.
7. Effective saturation values based on measured water contents above the water table in a neutron access tube located at a radial distance of about 5 m from the center of the pumping well lie very close to our pumping test derived effective saturation curve.

## ACKNOWLEDGMENT

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