

**Anomalous Transport in Heterogeneous Media with Sharply Contrasting Properties:
The Role of Diffusive Barriers – 11142**

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ABSTRACT

A key feature that has an extreme effect on the patterns of contaminant transport in fractured rocks is sharply contrasting properties of geological media. Sometimes contrasting media can be treated as two interacting subsystems of a high and a low permeability. We study analytically contaminant transport throughout a high permeability medium (fracture) with a diffusive barrier due to the localization of a contaminant source inside a low-permeability medium (matrix). Transport in the fracture is provided by advection and diffusion, whilst transport in the matrix is caused by diffusion only. We mainly focus on insulating properties of the diffusive barrier. It is shown that these properties are the exponential attenuation of the source power, retardation of the contaminant plume size spreading, and modification of the concentration distribution at large distances. Non-classical behavior of the contaminant concentration is observed. In particular, four different anomalous transport regimes take place, and the change of the regime occurs in both time and space.

INTRODUCTION

One of the major issues of nuclear power engineering is the management of radioactive waste management. High-level waste (HLW) is exceedingly dangerous, and its deep geological disposal has been internationally adopted as the most effective approach to assure long-term, safe disposition. Geological repository systems consist of series of engineered and natural barriers. The latter are expected to isolate HLW from the biosphere if engineered barriers fail. So it is extremely necessary to provide reliable assessments of insulating properties of geological media which are highly heterogeneous. However transport phenomena in non-uniform media have not been studied sufficiently enough at the moment. As known, heterogeneity may lead to the anomalous behavior of the tracer concentration. It is commonly assumed that if the time dependence of the root-mean-square displacement of the tracer particle has a form $R(t) \propto t^\gamma$, with $\gamma \neq 1/2$ [1-3], then anomalous diffusion takes place. We have subdiffusion for $\gamma < 1/2$ and superdiffusion for $\gamma > 1/2$. In addition, a crossover between different transport regimes can be observed [4,5]. Various approaches have been developed to describe non-classical transport, among which are the continuous time random walks (CTRW) [6] and aging CTRW model first introduced in the context of diffusion in glasses [7], the renormalization group method [8] initially devised within particle physics, and the fractional Fokker-Planck equation (FFPE) [9,10] and others. Some of them are purely mathematical and, a lack proper physical interpretation, resulting in inconsistencies in the obtained results [11,12]. Therefore, we prefer considering simple physical models without any additional assumptions, and so we paid a lot of attention to regular heterogeneous sharp contrasting media [13-15,18,19], that can be treated as two interacting subsystems of high and low permeability, with the source located inside the high permeability region. The simplest realization of such media has been first studied by Dykhne [13]; for this reason later it was called the Dykhne's model. In the present paper we analyze contaminant migration in Dykhne's model with a diffusive barrier due to the localization of the contaminant source inside the low-permeability region. Diffusive barriers (for example bentonite) are often used in geological repository systems, and many researches are devoted to the study of their insulating properties [20-23]. However, these properties may be underestimated without taking into account the anomalous behavior of the contaminant concentration arising in heterogeneous media.

PROBLEM FORMULATION AND BASIC RELATIONS

Consider contaminant transport in a heterogeneous system consisting of two parts (see Fig.1): a high-permeability medium I occupying a plane-parallel layer of a thickness a (fracture) and a low-permeability medium II, filling the rest of the space (matrix). Transport in the fracture is provided by advection with the velocity \vec{u} and diffusion, whilst transport in the matrix is caused by diffusion only. The coordinate z is chosen along the normal to the plane of the fracture and the coordinate x is along the advection velocity, so that $\vec{u}(z) = \begin{cases} \{u, 0, 0\} & \text{for } |z| < a/2 \\ 0 & \text{for } |z| > a/2 \end{cases}$. Let

$\vec{\rho}$ be a two-dimensional radius vector $\vec{\rho} = \{x, y, 0\}$. The diffusivity is

$$D(z) = \begin{cases} D & \text{for } |z| < a/2 \\ d & \text{for } |z| > a/2 \end{cases}, \quad D \gg d. \text{ Denote a contaminant concentration distribution by } c(\vec{\rho}, z; t).$$

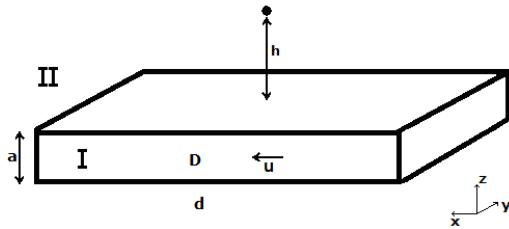


Fig.1 Dykhne's model geometry configuration

The advection-diffusion equation and boundary conditions have a form

$$\frac{\partial c}{\partial t} + \vec{u}(z) \nabla c - \text{div}(D(z) \nabla c) = 0, \tag{Eq. 1}$$

$$c(\vec{\rho}, z; t) \Big|_{z=\pm a/2-0}^{z=\pm a/2+0} = 0, \quad -D(z) \frac{\partial c(\vec{\rho}, z; t)}{\partial z} \Big|_{z=\pm a/2-0}^{z=\pm a/2+0} = 0. \tag{Eq. 2}$$

The contaminant source is assumed to be located inside the matrix at a distance h from the fracture and given by the initial condition

$$c(\vec{\rho}, z; 0) = N_0 \delta(z - h - a/2) \delta(\vec{\rho}). \tag{Eq. 3}$$

Taking the Fourier transformation of the (Eq. 1) with respect to $\vec{\rho}$ and Laplace transform with respect to t , we get

$$\left(p + i\vec{u}\vec{k} + Dk^2 - D \frac{\partial^2}{\partial z^2} \right) c_{p\vec{k}} = 0 \quad \text{for } |z| < a/2, \tag{Eq. 4}$$

$$\left(p + dk^2 - d \frac{\partial^2}{\partial z^2} \right) c_{p\vec{k}} = N_0 \delta(z - h - a/2) \quad \text{for } |z| > a/2, \tag{Eq. 5}$$

where p, \vec{k} is the Laplace and two-dimensional Fourier variables, respectively.

Further we call the particles located inside the fracture as active and our aim is to analyze

their concentration distribution given by $n(\vec{\rho}; t) = \int_{-a/2}^{a/2} dz c(\vec{\rho}, z; t).$

Integrating (Eq. 4) over z , we obtain

$$(Dk^2 + i\bar{u}\bar{k} + p)n_{p\bar{k}} + q_{p\bar{k}} = 0, \quad (\text{Eq. 6})$$

here $q_{p\bar{k}} = -D(z) \frac{\partial c_{pk}}{\partial z} \Big|_{z=-a/2}^{z=a/2}$ is the Fourier-Laplace transform of the flux density.

To find $q_{p\bar{k}}$ we solve (Eq. 5) with boundary conditions (Eq. 2)

$$q_{p\bar{k}} = n_{p\bar{k}} \sqrt{(p + dk^2)/t_1} - N_0 e^{-2\sqrt{t_h(p+dk^2)}}, \quad (\text{Eq. 7})$$

where $t_h = h^2/4d$, $t_1 = a^2/4d$.

Using (Eq. 6), (Eq. 7), we find the active particles concentration in the Fourier-Laplace space

$$n_{p\bar{k}} = \frac{N_0 e^{-2\sqrt{(p+dk^2)t_h}}}{Dk^2 + i\bar{u}\bar{k} + p + \sqrt{(p + dk^2)/t_1}}. \quad (\text{Eq. 8})$$

Consider the problem with the initial condition given by substitution $\delta(z - h - a/2) \rightarrow \delta(z)$ in (Eq. 3) that corresponds to the case where the source is located inside the fracture (this problem has been studied in [19]). The active particles concentration per unit area $n_{p\bar{k}}^*$ can be found by solving equation (Eq. 1) in the Fourier-Laplace space or using results of the work [19]. So we get

$$n_{p\bar{k}}^* = \frac{N_0}{Dk^2 + i\bar{u}\bar{k} + p + \sqrt{(p + dk^2)/t_1}}. \quad (\text{Eq. 9})$$

Comparing Eqs. (Eq. 8) and (Eq. 9), and then applying the inverse Fourier-Laplace transformation to function (Eq. 8), we obtain the relation

$$n(\bar{\rho}, t) = \int_0^t dt' \sqrt{\frac{t_h}{\pi(t-t')}}^3 \exp\left(-\frac{t_h}{t-t'}\right) n^*(\bar{\rho}, t'), \quad (\text{Eq. 10})$$

with

$$n^*(\bar{\rho}, t') = \frac{N_0 e^{i\bar{u}\bar{\rho}/2D}}{2\pi D} \int_{b-i\infty}^{b+i\infty} \frac{e^{pt'} dp}{2\pi i} \int \frac{d^2 \rho'}{4\pi dt'} e^{-\frac{(\bar{\rho}-\bar{\rho}')^2}{4dt'}} K_0(\rho'/Z(p)), \quad \text{Re } b > 0, \quad (\text{Eq. 11})$$

where $\rho' = |\bar{\rho}'|$ and K_0 is the MacDonald function (modified Bessel function of the second kind)

$$Z(p) = 2D/u \sqrt{1 + t_u(p + \sqrt{p/t_1})}, \quad (\text{Eq. 12})$$

here $t_u = 4D/u^2$.

Also we analyze two key parameters of the active particles transport such as the variance of the concentration distribution

$$R^2(t) = \frac{1}{N(t)} \int d^2 \rho \cdot n(\bar{\rho}, t) \rho^2, \quad (\text{Eq. 13})$$

and the active particles number

$$N(t) = \int d^2 \rho n(\bar{\rho}, t). \quad (\text{Eq. 14})$$

In the problem under consideration, the variance coincides with the contaminant plume size, which is a commonly used concept in the applied research.

MAIN RESULTS

Analysis of the concentration behavior

First note that for $t \ll t_h$ the function $n^*(\bar{\rho}, t')$ changes slightly with increase of t' , so using (Eq. 10) we obtain

$$n(\bar{\rho}, t) \cong n^*(\bar{\rho}, t). \quad (\text{Eq. 15})$$

As expected the concentration behaves as if the source is located in the fracture but not in the matrix. Since in this case the concentration behavior has already been found in [19], we focus on the opposite case

$$t \gg t_h. \quad (\text{Eq. 16})$$

The integrand exponent in (Eq. 10) decreases sharply with the increase of t' . Using a Taylor series expansion, we get

$$e^{-\frac{t_h}{t-t'}} \cong e^{-\frac{t_h}{t}} e^{\frac{t' t_h}{t^2}}. \quad (\text{Eq. 17})$$

So the contaminant source can be considered as continuously acting during the time interval equal $t_{\text{eff}} = t^2/t_h$ and locating inside the fracture but not in the matrix.

Using (Eq. 10), (Eq. 17), we obtain

$$n(\bar{\rho}, t) = \sqrt{\frac{t_h}{\pi t^3}} e^{-t_h/t} \int_0^\infty dt' e^{-st'} n^*(\bar{\rho}, t'), \quad s = t_h/t^2. \quad (\text{Eq. 18})$$

Comparing this equation with (Eq. 10), we find

$$n(\bar{\rho}, t) = N_0 \sqrt{\frac{t_h}{\pi t^3}} \frac{e^{\bar{u}\bar{\rho}/2D - t_h/t}}{2\pi D} \int \frac{d^2 \rho'}{4\pi dt} e^{-\frac{(\bar{\rho} - \bar{\rho}')^2}{4dt}} K_0(\rho'/Z(t_h/t^2)), \quad (\text{Eq. 19})$$

where $Z(t_h/t^2)$ is given by (Eq. 12).

For the distances $\rho \ll Z(t_h/t^2)$ we can use the approximation of Macdonald function:

$K_0(w) \approx -\ln w$, $w \ll 1$ in the above equation, so the concentration is

$$n(\bar{\rho}, t) \cong N_0 \sqrt{\frac{t_h}{\pi t^3}} \frac{e^{-t_h/t} e^{\bar{u}\bar{\rho}/2D}}{2\pi D} \cdot \begin{cases} \ln(Z(t_h/t^2)/\rho), & \sqrt{dt} \ll \rho \ll Z(t_h/t^2) \\ \ln(Z(t_h/t^2)/\sqrt{4dt}), & \rho \ll \sqrt{dt} \end{cases}. \quad (\text{Eq. 20})$$

For $\rho \gg Z(t_h/t^2)$ another approximation should be used

$$K_0(w) \approx e^{-w} \sqrt{\pi/2w}, \quad w \gg 1. \quad (\text{Eq. 21})$$

For $t \ll t_h$ both the exponential factor and $n^*(\bar{\rho}, t')$ of the integrand in (Eq. 10) have narrow peaks and the latter one decreases rapidly with increase of ρ . The exponential factor is more sharply peaked for $\rho/Z(t_h/t^2) \ll t_h/t$ with $Z(t_h/t^2)$ given by (Eq. 12). It follows from (Eq. 19) and (Eq. 21) that (Eq. 18) is valid for $\rho/Z(t_h/t^2) \ll t_h/t$ only.

Now consider the concentration behavior for $\rho/Z(t_h/t^2) \square t_h/t$. According to the results of our previous work [19], the function $n^*(\bar{\rho}, t')$ can be represented as follows

$$n^*(\bar{\rho}, t') \square \exp(-\Phi(\bar{\rho}, t')) \quad (\text{Eq. 22})$$

with $\Phi(\bar{\rho}, t) = (\bar{\rho}/R^*(t))^\alpha$.

Values of $R^*(t)$ and α is defined by the transport regimes obtained in [19].

Using (Eq. 22) and substituting $t' \rightarrow t - t'$, and then applying a Taylor series expansion for $\Phi(\bar{\rho}, t - t')$ around the point $t' = 0$, we obtain

$$n(\bar{\rho}, t) \square \int_0^\infty \sqrt{\frac{t_h}{\pi t'^3}} \cdot \exp\left(-t_h/t' - \Phi(\bar{\rho}, t) - t' \frac{\partial \Phi(\bar{\rho}, t)}{\partial t}\right) dt'. \quad (\text{Eq. 23})$$

Taking advantage of the saddle-point method, we get

$$n(\bar{\rho}, t) \square \exp\left(-2\sqrt{t_h} \cdot \frac{\partial \Phi(\bar{\rho}, t)}{\partial t} - \Phi(\bar{\rho}, t)\right). \quad (\text{Eq. 24})$$

This expression describes the concentration distribution at large distances (termed as concentration tails), namely, remote stages of the concentration tails as the first stage of the tails is given by (Eq. 19).

Transport regimes

There are several cases that differ from each other in their relations between the characteristic times t_u, t_1, t_h, t_2 , where $t_2 = t_1(D/d)^2$ also obtained in [19], so we consider the most interesting ones: 1. $t_u \square t_1 \square t_h$, 2. $t_1 \square t_h \square t_u, t_2$. First, note that advection affects the contaminant transport for $t \square \sqrt{t_u t_h}$. The reverse particle flux to the matrix becomes significant as $t \square \sqrt{t_1 t_h}$. It is clear that advection influences the contaminant migration transport in the case 1 and doesn't influence it in the case 2 since $t \square t_h$.

1. $t_u \square t_1 \square t_h$

Depending on the considered time interval, we can use different approximations for $Z(p)$ given by (Eq. 12).

As shown below, for $Z(t_h/t^2) \square \rho \square Z(t_h/t^2)t_h/t$ expression (Eq. 19) describes the main body concentration as well as a first stage of the concentration tail.

If $Z(t_h/t^2) \square \sqrt{dt}$, then using (Eq. 19), we get the slow classical diffusion

$$n(\bar{\rho}, t) \square \exp\left(-\frac{\rho^2 + h^2}{4dt}\right) \quad (\text{Eq. 25})$$

- 1.1 $t \square \sqrt{t_u t_h}$

Here we have $Z(t_h/t^2) \cong \sqrt{Dt^2/t_h}$, so (Eq. 19) gives

$$n(\bar{\rho}, t) \cong N_0 \frac{1}{2\sqrt{2\pi Dt}} \left(\frac{\sqrt{Dt_h}}{\rho} \right)^{1/2} \exp\left(-\frac{t_h}{t} - \frac{\rho}{\sqrt{Dt^2/t_h}} \right). \quad (\text{Eq. 26})$$

As follows from this expression, the contaminant plume size is

$$R(t) \square \sqrt{Dt^2/t_h}. \quad (\text{Eq. 27})$$

$$1.2 \sqrt{t_u t_h} \square t \square \sqrt{t_1 t_h}$$

Using the approximation $Z(t_h/t^2) \cong 2D/u(1+t_u t_h/2t^2)$, which is valid for this time interval, and (Eq. 19), we get

$$n(\bar{\rho}, t) \square \exp\left(-\frac{t_h}{t} - \frac{\rho}{ut^2/t_h} - \frac{\rho}{ut_u}(1-\cos\varphi) \right). \quad (\text{Eq. 28})$$

$$\text{where } \cos\varphi = \frac{(\bar{u} \cdot \bar{\rho})}{|\bar{u}||\bar{\rho}|}.$$

Thus, we have

$$R(t) \square ut^2/t_h. \quad (\text{Eq. 29})$$

$$1.3 t \square \sqrt{t_1 t_h}$$

Here $Z(t_h/t^2) \cong 2D/u\left(1 + \frac{t_u}{2}\sqrt{t_h/t_1 t^2}\right)$. It follows from (Eq. 19) that

$$n(\bar{\rho}, t) \square \exp\left(-\frac{t_h}{t} - \frac{\rho}{ut\sqrt{t_1/t_h}} - \frac{\rho}{ut_u}(1-\cos\varphi) \right). \quad (\text{Eq. 30})$$

and so the contaminant plume size is

$$R(t) \square ut\sqrt{t_1/t_h} \square ut. \quad (\text{Eq. 31})$$

It should be noted that the contaminant plume size can be obtained without solving the transport equation (Eq. 1) by means of the effective time concept and the results from [19]. We show this by considering the case 2. $t_1 \square t_h \square t_u, t_2$. In this case advection doesn't affect the contaminant transport, so the contaminant plume size can be expressed through the plume size of the problem [19] $R^*(t)$ as following $R(t) \square R^*(t^2/t_h)$. Recall some results of that work for $t_u \square t_1$

$$R^*(t) = \begin{cases} \sqrt{4Dt}, & t \square t_1 \\ \sqrt{4D\sqrt{t_1 t}}, & t_1 \square t \square t_2. \\ \sqrt{4dt}, & t \square t_2 \end{cases}$$

Substitution t^2/t_h for t in the above expression gives the plume size of the diffusive barrier problem

$$R(t) \square \begin{cases} \sqrt{Dt^2/t_h}, t \square \sqrt{t_1 t_h} \\ \sqrt{Dt\sqrt{t_1/t_h}}, \sqrt{t_1 t_h} \square t \square t_h \end{cases}. \quad (\text{Eq. 32})$$

Here we have only two regimes since $t_2 \square t_h$.

For $t_u \square t_1 \square t_h$ advection is significant, so $R(t) \square \max\{R^*(t^2/t_h), X^*(t^2/t_h)\}$, where $X^*(t)$ is first moment of the concentration distribution obtained in [19].

Thus, we find the contaminant plume size and concentration distribution for different time intervals that determines transport regimes specific for the diffusive barrier problem. All of the regimes (Eq. 26), (Eq. 28), (Eq. 30), (Eq. 32) have the retardation factor $t/t_h \square 1$, so for $t \square t_h$ the diffusive barrier results in the exponential attenuation of the source power and retardation of the plume size spreading.

Note that contaminant particles accumulate around the upper boundary between the matrix and the fracture, and the concentration distribution over z is highly inhomogeneous for $t \square \sqrt{t_0 t_h}$, where $t_0 = a^2/4D$. This inequality can be obtained as follows: particles accumulate in a layer with a thickness of $\delta z \square \sqrt{Dt_{eff}}$ due to the effective source action during the time interval t_{eff} .

The concentration is not homogeneous over z if $\delta z \square a$, hence $\sqrt{Dt^2/t_h} \square a$ and $t \square \sqrt{t_0 t_h}$.

For these times only two regimes occurs: $R(t) \square \sqrt{Dt^2/t_h}$, $R(t) \square ut^2/t_h$ since $\sqrt{t_1 t_h} \square \sqrt{t_0 t_h}$.

Concentration tails

Another consequence of the diffusive barrier presence is a modification of concentration tails comparing with those found in the problem without barrier [19].

Consider concentration tails for the case 1. $t_u \square t_1 \square t_h$. Depending on the time interval, the first stage of the tail is given by (Eq. 26), (Eq. 28), (Eq. 30). For $\rho \square Z(t_h/t^2)t_h/t$ the corresponding remote stages of the tails, the concentration is determined by (Eq. 24) and also varies with the time interval. Further, we obtain the results for one time interval only $t \square \sqrt{t_1 t_h}$, because for others the concentration can be found in a similar way. We show that concentration tails have a multistage structure: they consist of several parts, for each of them the concentration behavior being different.

If $Z(t_h/t^2)t_h/t \square \rho \square ut$, then $\Phi(\bar{\rho}, t) = (\rho/R(t))^2$, $R(t) = \sqrt{4u^2 t_1 t}$, so we get

$$n(\bar{\rho}, t) \square \exp\left(-\rho\sqrt{\frac{t_h}{u^2 t_1 t^2} - \frac{\rho^2}{4u^2 t_1 t}}\right). \quad (\text{Eq. 33})$$

If $\rho \square ut$, then $\Phi(\bar{\rho}, t) = (\rho/R(t))^2$, $R(t) = \sqrt{4Dt}$ and

$$n(\bar{\rho}, t) \square \exp\left(-\rho\sqrt{\frac{t_h}{Dt^2} - \frac{\rho^2}{4Dt}}\right). \quad (\text{Eq. 34})$$

Thus, for $t \square \sqrt{t_1 t_h}$ the concentration tail consists of three stages defined by (Eq. 30), (Eq. 33), (Eq. 34).

Active particle number

The time dependence of the active particle number $N(t)$ is given by (Eq. 14). Using (Eq. 14) and (Eq. 19), we find

$$N(t) = N_0 \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{dp}{2\pi i} \frac{e^{-2\sqrt{p}t_h + pt}}{p + \sqrt{p/t_1}}. \quad (\text{Eq. 35})$$

For $t \ll t_1$ the second denominator term of above expression should be neglected, so the integrating gives

$$N(t) \cong N_0 \text{erfc}(t_h/t). \quad (\text{Eq. 36})$$

For $t \ll t_h$ the error function $\text{erfc}(w) = \frac{2}{\sqrt{\pi}} \int_w^{\infty} e^{-v^2} dv$ can be approximated as $\text{erfc}(w) = \frac{e^{-w^2}}{w\sqrt{\pi}}$

since $w = t_h/t \ll 1$. Thus we have

$$N(t) = \frac{N_0}{\sqrt{\pi t_h/t}} e^{-t_h/t}. \quad (\text{Eq. 37})$$

For $t \ll t_1$ the second denominator term is much greater than the first one, so we find

$$N(t) \cong \frac{N_0}{\sqrt{\pi t/t_1}} e^{-t_h/t}. \quad (\text{Eq. 38})$$

This function is influenced by two factors: action of the effective source and diffusive particles flux from the fracture into the matrix. For $t \ll t_h$ the active particles number increases with time due to the influence of the second factor, then it reaches the maximum at $t = 2t_h$ and decreases as $N(t) \propto t^{-1/2}$ because of the first factor.

CONCLUSIONS

The contaminant transport through a regularly heterogeneous sharply contrasting medium with a diffusive barrier has been studied analytically. This medium is assumed to be consisting of two interacting subsystems of high and low permeability. The diffusive barrier emerges due to the localization of the contaminant source in the low-permeability medium (matrix) far away enough from the high-permeability region (fracture).

We have analyzed a concentration distribution of the particles located inside the fracture (active particles) for different time and distance intervals. It has been shown that for $t \ll t_h$ the source can be considered as acting during the time interval equal $t_{\text{eff}} = t^2/t_h$ and locating inside the fracture but not in the matrix. Depending on the time interval, we have observed one of several transport regimes that are specific for the diffusive barrier problem:

$R(t) \sim ut^2/t_h$, $R(t) \sim ut\sqrt{t_1/t_h}$, $R(t) \sim t\sqrt{D/t_h}$, $\sqrt{Dt}\sqrt{t_1/t_h}$, where $R(t)$ is the contaminant plume size. A crossover between different transport regimes is observed.

The concentration distribution at large distances (concentration tails) has an exponential form and a multistage structure as in our previous works [15-19]. Thus the change of transport regime occurs in both time and space.

The evolution of the active particle number has been found: $N(t) = N_0 \cdot \operatorname{erfc}(t_h/t)$ for $t \ll t_1$

and $N(t) = \frac{N_0}{\sqrt{\pi t/t_1}} e^{-t_h/t}$ for $t \gg t_1$. In the latter expression the first factor is provided by the

diffusive flux from the fracture, while the second is caused by the action of the effective source.

We emphasize that all observed transport regimes have a retardation factor of $t/t_h \ll 1$ as compared with the results of the problem without a diffusive barrier [19]. Also, the active particle number has an exponential attenuation factor. Thus, we conclude that the diffusive barrier results in:

1. The renormalization of the source power, namely, exponential attenuation of the power;
2. The retardation of the plume size spreading; 3. modification of the concentration tails.

Diffusive barriers are widely used, for example, bentonite is often considered as the best buffer and backfill material for deep geological repositories. Assessments of their insulating properties should be performed taking account anomalous transport behavior. Non-classical features of the contaminant transport may be used to make the repository more effective, in particular, by increasing the storage capacity without loss safety.

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