PRECISION OF THE ACCIDENTALS RATE IN NEUTRON COINCIDENCE COUNTING - 10475

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ABSTRACT

The quantification of Pu present, for example, as process hold-up in glove boxes or in waste containers may be accomplished by passive neutron coincidence counting (PNCC) to determine the Pu-240 effective mass which may then be related to the total Pu mass or Pu-239 mass through the isotopic composition. When multiplication can reasonably be taken as slight, assay results are normally based on the Reals coincidence rate since in contrast the specific Totals rate is sensitive to (α ,n) production which may be difficult to estimate accurately. The precision of the net Reals rate and hence the quality of the assay depends on how well the subtraction of the Accidental coincidence rate can be made. Three approaches are readily available with current standard shift register modules, these are the signal triggered (or measured) Accidentals rate, the rate calculated from the Totals event rate, and the rate estimated according to the so called fast accidentals sampling method.

The NDA literature contains practically no systematic information on when and how calculated Accidentals and fast Accidentals should be used to complement or replace the conventional signal triggered (measured) estimate. This work aims to redress this by making a systematic study, in the low efficiency domain typical of glove boxes and basic large volume waste assay systems. We discuss how good measurement control can be attained with shorter measurement times or lower efficiency when the best Accidentals treatment is used.

PNCC is widely used for waste assay and MC&A throughout the nuclear fuel cycle. Achieving highly accurate results in a timely and cost effective way is an important operational goal, especially in the planning of new facilities and in the context of He-3 gas shortage. This work looks at how data analysis techniques can contribute to this end.

We report a systematic study performed using Cf-252 sources measured between a pair of thermal neutron slab counters configured to achieve a detection efficiency ranging from about 2.5% to about 9.6%. Various coincidence gating structures and rates were examined. We discuss the findings in terms of practical performance advantage and, in the case of Totals and Reals compare the observed precisions to simple theoretical estimates.

INTRODUCTION

The spontaneous fission of Pu liberates a burst of neutrons. The released number is a random variable, typically between 0 and 8 and follows a characteristic probability distribution. The neutrons which emerge from the item enter the moderator of the surrounding He-3 proportional counter based thermal neutron detector where, in simple terms, they are rapidly slowed down (with a μ s or two) and then migrate as thermal neutrons until they either leak from the system, captured parasitically or are detected (the characteristic 1/e time for this diffusive behavior being typically of the order of 50 μ s to 100 μ s). The detected events from a given spontaneous fission are therefore detected over a period of time commensurate with the lifetime of neutrons in the system. The slowing down time is short compared to the characteristic lifetime and if we plot vs time the detection time distribution of neutrons relative to each detected thermal neutron one obtained a dieaway profile which can, sufficient for the present purposes, be approximated by a single exponential functional form.

The shift register circuit, see Bondar et al [1] for the history and action, is one means of extracting the time correlated signal from the pulse train. One is interested in the time correlation because it can be appreciated that the number of coincident pairs of events is related to the spontaneous fission rate. Conceptually the circuit works by opening a coincident gate of width, Tg, of the order of the dieaway time, called the (R+A)-gate, a short period known as the predelay, Tp, after every detected event. The number of events in the (R+A)-gate are scaled and the rate is obtained from by dividing the aggregate number observed by the duration of the data acquisition. It contains the genuine or Reals coincidence events, that is those neutrons born from the same fission and detected close in time governed by the characteristic neutron transport properties of the assay chamber (sometimes influenced by the item being studied). To obtain the random, chance or Accidentals contribution to the (R+A)-gate a second gate of equal duration known as the A-gate is opened after a long delay T_L. The long delay is chosen to be many times longer than the dieaway time so that any events which fall inside it cannot plausibly have come from the initiating or trigger event. The signal triggered Accidentals tally obtained in this way we shall refer to as the measured Signal Triggered A-rate, A_{ST}, and the net Reals rate R is obtained from:

$$R_{ST} = (R+A)_{Meas} - A_{ST}$$
 (Eq.1)

If we denote the Total (or gross) event rate by T after a little thought we can appreciate the A-rate may also be calculated in straightforward way from $A_{Calc}=T^2.T_g$. Written this way, the Accidentals rate is referred to as the Calculated Accidentals, hence A_{Calc} . This form can be appreciated by recognizing that on the average the A-gate is opened T times per second and the contents at each sampling are expected to be $T.T_g$. Thus we have an alternative expression for R:

$$R_{Calc} = (R+A)_{Meas} - A_{Calc}$$
 (Eq.2)

As defined above we can also appreciate that there is nothing special about A_{ST} . By using different values of T_L , conceptually both before and after the incident trigger event one can create different estimates of equivalent statistical power. In addition one can ask why the A-gate is opened by the incoming pulse train and not by some other scheme such as an artificially imposed psueudo-random pattern or according to a contiguous non-overlapping sequence of gates. An alternative scheme implemented on some designs of shift register electronics is periodic sampling. When performed at the clock frequency of the unit the pulse train is interrogated at a rate far greater than the sustained

incoming event rate and so is referred to as Fast Accidental Sampling (FAS) [2]. Each segment of the pulse train may be sampled multiple times but on the average the expectation value will be correct. Based on FAS we therefore have a third expression for R given by:

$$R_{FAS} = (R+A)_{Meas} - A_{FAS}$$
 (Eq.3)

The aim of this work was to systematically investigate which of the three schemes listed above produced Reals rates with the best precision over a range of conditions of practical interest. In addition we wanted to set the ground work for checking the theoretical dependences developed by Dytlewski et al [3] since very little attention has been given to this in the literature.

EXPERIMENTAL SET-UP

The experimental data used in this work was collected using two Canberra He-3 neutron slab Counters. The setup is pictured in Figure 1. The slabs each contain 6 He-3 proportional tubes embedded in highdensity polyethylene (HDPE) and each slab is serviced by an Amptek® JAB-01 based chargeamplifier-discriminator board. The two TTL signal are physically summed (i.e. ORed) to result in a train of 50 ns wide TTL pulses. The pulse train is fed into the latest generation Multiplicity Shift Register (MSR) electronics JSR-15 [4]. The high voltage (HV) setting was 1680 V slightly above the knee of the operating characteristic. Data was collected using both NDA 2000 and INCC 5.02 software packages on a standard Personal Computer.



Figure 1 Two slab neutron counters in a "V" position ($\epsilon \sim 5.4\%$) and a Cf-252 source in the middle of the detectors inserted in a cardboard box.

The JSR-15 MSR, shown in Figure 2, is a portable, fully computer-controlled, neutron analyzer that functions in the Canberra 2150 Multiplicity Mode. It is a specialized pulse counter used primarily to count neutron events originating in neutron detection instruments. While the counter can be used to count any TTL or differential input pulse train with 20 ns pulse pair resolution, its ability to record time correlated events and the multiplicity distributions of these events renders it suitable for counting

neutron events in the nuclear fields of material safeguards, waste assay and process monitoring and control. Compared to earlier units Fast Accidentals Sampling is implemented and deeper registers have been used to accommodate the high sustained rates without overflows. It complies with the International Atomic Energy Association (IAEA) neutron coincidence counting requirements [4].



Figure 2 JSR-15 Hand Held Multiplicity Shift Register. There is a single shifter register input and two extra scalers. The JSR-15 weight is 1.7 kg and its dimensions are 254x203x32 mm

Various neutron efficiencies were achieved using the "V" geometry by placing the source along the median. Reducing the angle between the two slabs a little bit and keeping the source at the approximate same position gives a ~9.6% efficiency. Moving the source along this median (still on the lightweight cardboard pedistal) yields efficiencies ranging from ~1% to ~9.6%. Four categories of experiments were performed as listed in Table 1.

Cf-252 Sources ID	Efficiency [%]	Gate Widths [µs]
01-1; 04-1	2.5	8, 16, 32, 64, 80, 96, 128, 256
01-1; 04-1	5.4	32, 64, 128
01-1; 04-1	9.6	32, 64, 128

Table 1 Gate width settings for each efficiency configuration for sources (01-1 and 04-1). The emission rate for Cf-252 source 01-1 is \sim 22,653 n/sec (as of 08/05/2009) and for source 04-1 the emission rate is \sim 52,2341 n/sec (as of 08/05/2009). The cycle time was set to 10 sec and the assay time to 3600 sec. For each of the above configurations data were taken in both Signal Triggered and Fast Accidentals modes. Calculated Accidentals was evaluated off-line.

Another set of measurements was performed using source 01-1 and Signal Triggered mode at efficiency settings of: 1.25, 2, 3.08, 4, 5.04, 6.35, 7.44, 8.41 and 9.3 % with the gate width was set to 64µs (determined to be close to the optimum setting for these slabs).

ANALYSIS AND RESULTS

Suppose for a moment that we simplistically, as a matter of mathematical convenience, treat the number of events in the (R+A)-gate and the A-gate as if they were the result of a pure matched pair Poisson counting experiment. Denote the rates by (R+A), R and A respectively and the data acquisition (cycle) time by t_{cycle} . Then the standard deviation (root variance) on R is given by:

$$\sigma(R) = \frac{1}{t_{cycle}} \cdot \sqrt{(R+A)t_{cycle} + At_{cycle}}$$
(Eq.4)

Now, as is often the case, (e.g. for neutron efficiency $\epsilon <<1$, leakage self multiplication $M_L \sim 1$, random-to-(SF,n) production ratio $\alpha >1$, say) the genuine Reals rate R<<A and so we see that the estimation of the variance collapses to roughly:

$$\sigma(R) = \sqrt{\frac{2.A}{t_{cycle}}} = \sqrt{2} \cdot \sigma(A)$$
 (Eq.5)

subject to the assumptions and approximations stated. Thus, we recognize that the precise determination of the Accidentals rate, A, is vital to obtaining good precision on the net Reals rate, R, in the high rate domain and this is the driver for operating with a short gate width (to reduce the chance of a events falling into the gate by chance) which in turn is the motivation to design counters with short dieaway times so that the genuine coincidence signal is spread out only over a short period of time (meaning that when a short gate is used genuine coincidence events are not disproportionately discarded). Of course one cannot operate with an arbitrarily short gate width because then the signal will also eventually vanish.

In the case of FAS the variance on A can now, in principle, be reduced to a level where it is small compared to our estimate of the variance on (R+A) so that Eq.4 collapses, in the limit (and still based on the Poisson assumption), to:

$$\sigma(R) = \sqrt{\operatorname{var}(R+A) + \operatorname{var}(A)} \approx \sqrt{\operatorname{var}(R+A) + 0} \approx \frac{1}{t_{cycle}} \sqrt{(R+A) \cdot t_{cycle}} = \sqrt{\frac{(R+A)}{t_{cycle}}}$$

$$\approx \sqrt{\frac{A}{t_{cycle}}}; \quad In \text{ the limit } R \ll A.$$
(Eq.6)

which we recognize as being a factor of sqrt(2) smaller (compared to Eq.5 that is) in the high rate domain being considered.

So, we can appreciate that the promise behind the FAS scheme is improved precision, at least over a certain dynamic range, the benefit of which needs to be established for a given application. In practice when the MSR clock speed is very much greater that the count rate the incoming pulse train is massively over sampled. This further violates the Poisson assumption upon which our analysis thus far is based. That is to say each event may be used many times in forming the estimate of A. This is not to say that the estimate of A will be biased however. On the other hand FAS can not invent more

information carriers than are present and so the statistical power of the method is naturally limited by the data steam. The over sampling process may well add correlations which are to our knowledge not kept track of explicitly.

With the way in which the FAS scheme works and established, let us now consider how one may in addition calculate the Accidentals rate. Each incoming neutron event triggers the inspection of the A-gate. If the incoming rate is denoted by T that means in an experiment of cycle duration t_{cycle} we have N=(T.t_{cycle}) inspections. The expected number of counts, C, in the A-gate is given by the product of the counting rate and the duration of the gate, that is C=(T.T_G)=N.(T_G/ t_{cycle}). Our estimate of the expected number of counts, A. t_{cycle} , observed in the A-gate is thus N.C – the product of the number of times we open the gate and the expected number of counts in the gate - so that the A count rate over the period based on the observed Totals rate, T, is expected to be:

$$A_{Calc} = \frac{1}{t_{cycle}} N \cdot C = \frac{1}{t_{cycle}} (T \cdot t_{cycle}) \cdot (T \cdot t_G) = T^2 \cdot T_G = N^2 \frac{T_G}{t_{cycle}^2}$$
(Eq.7)

Now, provided the Total event rate T is not subject to significant fluctuation over the course of the data acquisition time, t, due to, for example, changing cosmic ray intensity (e.g. atmospheric pressure) or plant conditions (the movement of materials in the vicinity of the counter) we expect Eq.7 to yield a fair estimate of the average A-rate and moreover to have equivalent statistical power as the estimate which would be obtained from the FAS scheme by virtue of the fact that it is based on using all N pieces of information gathered, N-being the total number of events observed and therefore being the sum total of information carries we have to bring to bear on our problem. Note that in using N as the primary variable, the correlation between N and C is automatically taken into account. Treating N and C as separate variables would call for consideration of the covariance between them in the calculation of A_{calc} . The issue in considering A_{FAS} and A_{ST} , in comparison, is to do with the sampling as opposed to using the full pulse train.

In using A_{calc} to correct (R+A) cycle by cycle to form the net Reals rate R, the issue becomes whether A_{calc} can be determined with far superior precision than A_{ST} (the simple event triggered estimate of the A-rate) over the dynamic range and cycle structure of practical interest so that a overall improvement in assay precision can be obtained. The easiest way to address this is to mock-up a passive neutron coincidence counter (e.g. a simple slab in the case of addressing a glove box monitor) and make a systematic series of measurements covering a range of count rates (e.g. by changing the Cf source not the efficiency (position)), cycle time and coincidence gate-width.

Inspection of Eq.7 would suggest that if we are in a regime where we can approximate Totals counting as a Poisson process (and T_G and t_{cycle} may be taken as very well known) then the standard deviation may be approximated as:

$$\sigma_{A_{Calc}} = 2 \cdot (T \cdot T_G) \sqrt{\frac{T}{t_{cycle}}}$$
(Eq.8)

or, in terms of the relative standard deviation:

 $\frac{\sigma_{A_{Calc}}}{A_{Calc}} = 2 \cdot \sqrt{\frac{1}{T \cdot t_{cycle}}}$ (Eq.9)

 T_G is typically of the order of 16µs to 128µs and t_{cycle} >1s. Increasing t_{cycle} reduces $\sigma_{A_{Calc}}$ in inverse proportion to the root time. The ratio plots of the relative uncertainty in the Accidentals to the relative uncertainty in the Totals (using Cf-252 source 01-1 in a 5.4% efficiency configuration) are shown in Figure 3 for the measured Signal Triggered Accidentals and in Figure 5 using the Fast Accidentals Sampling (FAS) Accidentals. The plots were generated using the acquired 360 cycles of 10-sec. One can divide them into 10 groups; each with 36 statistically independent 10-sec-cycles. For each group we can estimate the Accidentals and Reals averages and uncertainties by taking the standard deviation. The standard deviation of the 10 groups estimates will serve as an uncertainty on the data points. Also we present the comparison between the relative uncertainties in the Reals rate using the ST Accidentals (in Figure 4 using ST Accidentals and in Figure 6 using FAS Accidentals). We conclude that the Calculated T²T_G Accidentals decreases the uncertainty in the Reals if compared with the ST generated Reals. However, in the dynamic range we used, the FAS gave consistent results if compared with the T²T_G calculated values. This conclusion was verified using a higher dynamic range (with Cf-252 source 04-1) and various efficiencies ranging from 2.9% to 9.6%. The results are presented in detail in Appendix A.



Figure 3 The Ratio of the Accidentals relative uncertainty to the Totals relative uncertainty using source 01-1 as a function of the gate width. The Accidentals are estimated using both the measured Signal Triggered and calculated Accidentals. Each data point is an average estimate from the 360 10-sec-cycles. The efficiency of this configuration was 5.4%.





Figure 4 Plot of the Reals relative uncertainty using source 01-1 as a function of the gate width. The Reals are estimated using both the measured Signal Triggered and Calculated Accidentals. Each data point is an average of the 10 groups (36 cycles of 10-sec) estimates. The efficiency of this configuration was 5.4%.



Figure 5 Plot of the Ratio of the Accidentals relative uncertainty to the Totals relative uncertainty using source 01-1 as a function of the gate width. The Accidentals are estimated using both the FAS and Calculated Accidentals. Each data point is an average estimate of the 360 cycles of 10-sec. The efficiency of this configuration was 5.4%.



Figure 6 Plot of the Reals relative uncertainty using source 01-1 as a function of the gate width. The Reals are estimated using both the FAS and Calculated Accidentals. The efficiency of this configuration was 5.4%.

COMPARISONS WITH THEORY

For a single counting cycle of duration t_{cycle} , uncertainties on the net Totals and the Reals rates in Neutron Coincidence Counting (NCC), based on a simple exponential dieaway response in the high count rate regime (A>>R) where dead time perturbations are not too severe, are expected to be approximated by the following semi-empirical relations [3].

$$\sigma_{T} = w_{T} \cdot \sqrt{\frac{T}{t_{cycle}}}$$
(Eq.10)
$$\sigma_{R} = w_{R} \cdot \sqrt{\frac{(R+A)+A}{t_{cycle}}}$$
(Eq.11)

where the weighting factors are given by:

$$w_T \sim \sqrt{1+2.\frac{(R/f_d)}{T}}$$
 and $w_R \sim \sqrt{1+8.\frac{\left(1-\left(1-e^{-\frac{T_G}{\tau}}\right)\right)}{(T_G/\tau)}}.\frac{(R/f_d)}{T}$ (Eq.12)

where T_p , T_G and τ are the pre-delay, coincidence gate width and dieaway time respectively and f_d is the coincidence gate utilization factor. We recognize $(R/f_d)/T$ as the ideal Reals-to-Totals ratio for perfect gating. We shall concentrate on the behavior of w_R since uncertainties on the Reals alone are often used in waste assay applications via calibration curve to derive the mass value. Traditionally the gate width is set to about 1.257. τ in order to obtain the best fractional precision in the net Reals by operating close to the broad shallow minima in the plot of relative precision (σ_R/R) vs. (T_G/T_d) (the numerical value comes from treating the problem as a Poisson counting experiment although the choice of gate width is commonly made based on actual experimental plots).

We can re-write (R/f)/T in terms of basic nuclear parameters and properties of the assay chamber. For simplicity let us additionally assume that the ambient background rates can be neglected (inclusion complicates the algebra but the discussion is not affected). As already stated we are also assuming that

deadtime effects are not strong enough to significantly distort the statistics of the pulse train. Then, invoking the familiar point model expressions we have:

$$\frac{\left(\frac{R}{f_d}\right)}{T} \sim \frac{\varepsilon \cdot \left(\frac{v_{S2}}{2}\right)}{v_{S1}} \cdot \left(\frac{M \cdot \left[1 + \kappa \cdot (1 + \alpha) \cdot (M - 1)\right]}{(1 + \alpha)}\right) \text{ and } \kappa = \frac{v_{S1} \cdot v_{i2}}{v_{S2} \cdot (v_{i1} - 1)}$$
(Eq.13)

where ε is the neutron detection efficiency, v_1 and v_2 are the first and second factorial moments of the prompt spontaneous fission neutron emission multiplicity distribution respectively, M is the leakage multiplication factor, α is the random (dominated by (α,n)) to (SF,n) ratio, and $\kappa = v_{S1}/(v_{I1}-1).(v_{I2}/v_{S2})$ " is a function of basic nuclear data which happens to be rather insensitive to the exact nature of the problem taking on a value of approximately 2.17 in the case of Pu assays of common practical interest. Heuristically one can think of the ratio (R/f_d)/T as the parameter which quantifies the degree of correlation present in the pulse train. That is the correlated pairs to total number of events on the pulse train entering the shift register. A comparison between the Reals relative uncertainty, using source 01-1 as a function of the "V" configuration efficiency, and the expected value, using Eq. 12 is given in Figure 7. We note a significant difference in need of an explanation.



Figure 7 Plot of the Reals relative uncertainty using source 01-1 as a function of the "V" configuration efficiency. The measured Reals were generated using the ST technique.

CONCLUSIONS AND FUTURE WORK

Although countless thousands of PNCC assays must have been performed over the decades we are unaware of any extensive systematic studies that have been performed to evaluate the validity and accuracy of the approximate expression for σ_T and σ_R given above; for example studies which cover a

broad range of material types, a comprehensive range of assay times, and a representative spread of counting rates for a selection instruments in order to confirm the functional dependence of the w-factors introduced in [3].

However, in practice it is commonplace, and good practice, to divide the assay time into a series of shorter counting periods so that there are a reasonable (>15) number of cycles. A statistical analysis of the cycle data permits the data set to be checked for internal consistency and the uncertainties and correlations to be extracted from the fluctuations observed in the data. This makes estimating uncertainties from a single cycle moot. Normally both the standard deviation for the cycle and the standard error for aggregate data set are computed. As routinely practiced statistical analysis of the PNCC cycle data is an important step in checking the correct operation of the instrument throughout the data collection period and also in the detection and rejection of rare, spurious or anomalous events such as: electrical spikes, humidity related HV discharges, cosmic ray bursts and source movements in the vicinity during the assay. Statistical filters using the SDs (Standard Deviation) are used for this. We suggest evaluating the means rates from the summed data of all the valid cycles since the solutions are non-linear and use the scatter to evaluate uncertainties and correlations as needed. In general we would advocate calculating compound terms of interest directly rather than from preprocessed means. Bounding the precision on the final assay makes use of the SE (Standard Error).

This study quantifies the benefit of using T^2T_G calculated and FAS measured Accidentals for the dynamic range of interest and using Cf-252 sources under conditions where T is known to be under control.

An obvious extension of the work presented here is the quantification of the concluded effects using Am/Li sources (to bolster to component of uncorrelated events present on the pulse train) and higher efficiencies (representative of safeguards multiplicity counters, for example).

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Appendix A

Plots representing Relative Standard Deviation ratios RSD(A)/RSD(T) using source 01-1 with Signal Triggered (or FAS) Accidentals compared with calculated Accidentals.

(a) Signal Triggered.

(b) Signal Triggered. Efficiency=5.4%

(d) Signal Triggered. Efficiency=9.6%



12

Plots representing RSD(A)/RSD(T) using source 04-1 with Signal Triggered (or FAS) Accidentals compared with calculated Accidentals. **(b)** Signal Triggered. Efficiency=5.4% (d) Signal Triggered. Efficiency=9.6%



(a) Signal Triggered. Efficiency=2.5%

Plots representing the Reals relative uncertainty using source 01-1 with Signal Triggered (or FAS) Accidentals compared with calculated Accidentals.



14

Plots representing the Reals relative uncertainty using source 04-1 with Signal Triggered (or FAS) Accidentals compared with calculated Accidentals.



(a) Signal Triggered. Efficiency=2.5%