Comparison of Deadtime Correction Factors for Passive Neutron Multiplicity Counting of Correlated and Non-Correlated Neutron Sources - 9298

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ABSTRACT

Traditional deadtime correction methods for Passive Neutron Multiplicity Counting (PNMC) have been found to be potentially accuracy limiting at high average (or sustained) count rates and in the case where highly correlated rates occur over a short coincidence gate width (high instantaneous rates associated with high multiplicity bursts). It is costly to retrofit new hardware to existing assay systems to reduce the effects of deadtime, thus it is advantageous to develop and implement new deadtime correction algorithms as an alternative approach to ameliorate this problem. Future counter designs trend towards higher efficiencies and shorter dieaway times and hence these designs will also present the need for improved deadtime treatments as they will get applied to more demanding applications. For these reasons, deadtime correction techniques for PNMC are currently being re-visited by both the waste characterisation and safeguards communities in the nuclear industry.

A Monte Carlo approach has been established to simulate deadtime behaviour in PNMC systems and applied to this long standing problem. The form of the deadtime correction factor has been investigated for non-correlated (e.g. AmLi) neutron sources and will be extended to correlated (e.g. Cf-252) neutron sources. This paper addresses the practical correction method in each case. The aim of this work has been to aid research into the development of an improved and unified approach to deadtime correction for different multiplicity distributions. Simulation provides a convenient means to examine the range of applicability of current analytical models.

INTRODUCTION

Correlated neutron counting techniques such as Passive Neutron Coincidence Counting (PNCC), and PNMC using Multiplicity Shift Register (MSR) pulse train analysis are widely applied at nuclear fuel cycle facilities for the Non-Destructive Assay (NDA) of Pu. These techniques have applications in both waste management and safeguards measurements, including the assay of Pu metal, oxide, scrap, residues, waste and Pu oxide in excess weapons materials [1]. Correlated event rates are derived from the resultant multiplicity histogram and used to quantify mass values of spontaneously fissile materials. Extraction of correlated event rates (e.g. Doubles, Triples) is reliant on the application of accurate deadtime correction factors at high count rates. In these count rate regimes, it is important that deadtime correction factors themselves do not become the major accuracy limiting factor, compared to the counting precision.

Deadtime Challenges

There are several practical motivations driving an investigation into deadtime correction factors for PNMC. Evolution of counter design features, together with an extension of the technique to a greater range of applications and the assay of more demanding items, are likely to result in the need for improved deadtime treatments. Future generations of multiplicity counters and their applications will trend towards the following [2]:

- Increased neutron detection efficiencies (currently $\varepsilon \sim 50\%$)
- Shorter capture time distributions and hence reduced dieaway times (currently 30-40 μs)

- Higher sustained count rates as a result of increasing the range of masses of assay items e.g. ILW, high rate safeguards applications
- Assay of impure items with high (α, n) rates i.e. an increased ratio, α, of (α, n) neutrons to neutrons born in spontaneous fission. Increased induced fission as a result of self-interrogation.
- High self-leakage multiplication, M_L, resulting in long fission chains

A reduced counter dieaway time results in counter operation at shorter coincidence gate widths (reduced gate utilisation factor). For a given ε, higher instantaneous count rates will be imposed by this reduction in gate width, potentially with a corresponding increased item count rate (high sustained rate). High instantaneous count rates mean there is an increased likelihood of detecting a large number of events in a single coincidence gate width, hence detecting higher multiplicities of events resulting in the multiplicity histogram extending to high order. Consequently, an increased number of accidental correlations may be observed and deadtime corrections will need to be applied to extract the true correlated rates. However when the instantaneous counting rate is high, the uncertainties in the applied deadtime corrections can be the accuracy limiting factor in the derived count rates.

Since the underlying physical behaviour of deadtime to date has not yet been thoroughly investigated in PNMC, there is a corresponding physics motivation to investigate the analytical forms of the deadtime correction factors themselves to improve our understanding.

Monte Carlo Pulse Train Analysis

In a recent paper [3], a Monte Carlo approach to neutron pulse train analysis and the investigation of deadtime behaviour was established, based in essence on methods in [5] and [6]. This simulation approach has several merits; the first is that it allows the effects of deadtime to be directly applied to ideal neutron pulse trains, according to any chosen model. Deadtime behaviour and corresponding deadtime correction factors can then be investigated for a range of sources. In addition to this, input parameters are fully known, including counter operational parameters and the input system deadtime parameter. This method was used to investigate the change in the observed MSR data with increasing deadtime parameter i.e. the decrease in the measured Singles, Doubles and Triples rates. The method was also used to compare the relative effects of two familiar theoretical deadtime models – both paralyzable (updating) and non-paralyzable deadtime [7].

Recent investigations in [4] used this approach to study the effect of building pulse trains with different proportions of correlation on the observed rates. The traditional deadtime correction factors applied to Singles and Doubles counting were shown to be approximate and the level of approximation dependent on the degree of correlation on the pulse train.

Here, that methodology is extended to include additional calculations for a pure random pulse train. The form of the deadtime correction factor has been investigated for a non-correlated (i.e. random in time) neutron source and directly compared to the theoretical paralyzable model of deadtime. Later, the method may be extended to compare deadtime correction factors for correlated neutron sources such as Cf-252 and additionally review the applicability to highly multiplying items such as Pu.

Simulation work can be used to explore the basis of the often as factor 4 between Totals and Reals correction factor for traditional NCC deadtime correction methods. The relative merits of different deadtime schemes may be explored for an ideal detector as a precursor to system specific simulations.

SIMULATION METHOD

The radiation transport code MCNPX [8] was used to model an ideal ^{3}He neutron detector head with 4π geometric coverage. This ideal detection geometry was chosen to allow direct comparison of data to existing theory. Detector parameters were then chosen to maximise detection efficiency and shorten the capture time distribution in order to mimic the expected behaviour of future counter designs.

The detector was modelled with large radius 1000 m so that it could be considered to be self-moderating and hence ensure that the maximum number of neutron interactions led to neutron capture in the ^{3}He, resulting in a high efficiency, ε ~93%. The density of the fill gas was chosen to be 1.65 kg.m^{-3}. This density corresponds to a relatively high pressure of 13.5 atm at 300 K for the detector volume, resulting in a short dieaway time of 2.6 μs. In many respects this detector represents the ultimate ^{3}He based system.

MCNPX was used to model a random neutron source with timing characteristics representative of an (α, n) source e.g. AmLi commonly used for PNMC system characterisation. The Pu source was modelled as a 100 g sphere (radius 1.06 cm) of PuO_2 at an artificial density of 20 g.cm^{-3} (88.19 g total Pu mass) with isotopic composition 6% ^{240}Pu and 94% ^{239}Pu. The source was positioned at the centre of the detection geometry. Neutrons were sampled from an O (α, n) energy spectrum integrated over emission angles for 5.5 MeV α -particles tabulated in Jacobs, *et al* [9]. However for present purposes, both the source material and this energy spectrum are arbitrary since it is the random timing characteristics of the source that are of interest only.

Single, random neutrons are emitted from (α, n) reactions and therefore (in the absence of multiplication) no temporal correlations exist. Ideal neutron pulse trains were generated for a range of count rates using the method described in [3]. An assay time of 1000 s was chosen in each case, with 20 counting segments for statistical uncertainty analysis. Simulated MSR parameters of the pre-delay, gate width and long delay were chosen to be 0.3 μs, 3 μs and 200 μs, respectively. In units of the dieaway time these are in-line with field deployed systems.

RESULTS AND DISCUSSION

Comparison of random pulse train data to ideal paralyzable model

Ideal random pulse trains were generated and subsequently perturbed by the action of a deadtime i.e. several deadtime parameters were overlaid on the pulse train. Figure 1 shows the effect of overlaying a range of deadtime parameters on the simulated (akin to the measured or observed) Singles rate, S_m, for random pulse trains at count rates covering a range representative of practical interest. The ideal deadtime corrected Singles rate, S_c, is equal to the count rate calculated from the MSR analysis with the deadtime parameter, δ, set to zero. Both S_m and S_c are defined in units of s^{-1}, or Hz.

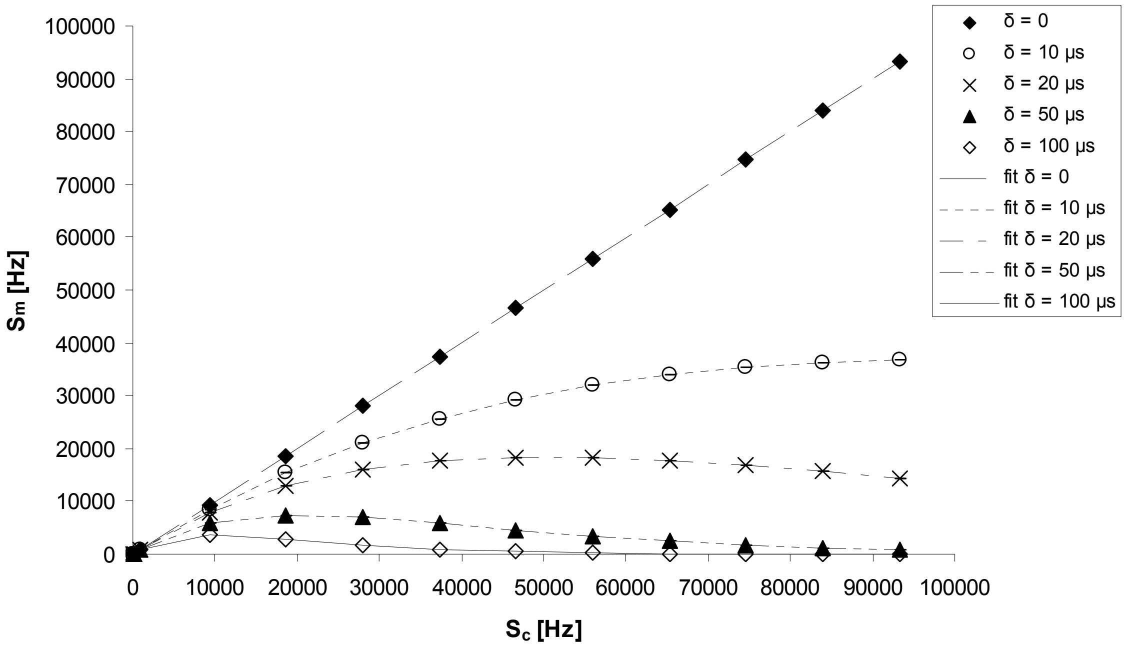

Fig. 1. Measured Singles rate, S_m against deadtime corrected Singles Rate, S_c for an uncorrelated neutron pulse train with paralyzable deadtime, δ. S_c may also be thought of as the 'true input count rate' presented to the ideal MSR.

The measured or true rates were fit to the transcendental equation for the paralyzable model for a true random (pure Poissonian) neutron source. For a random pulse train the measured (or observed) rate with paralyzable deadtime, S_m is related to the corrected (or true) rate, S_c through the following expression:

$$S_m = S_c \exp(-\delta \cdot S_c) \qquad \text{(Eq. 1)}$$

where δ is the system deadtime parameter.

Fitting the simulated data to the theoretical relationship confirmed that the effects of deadtime have been correctly overlaid on the pulse train according to the paralyzable model applied. The use of an uncorrelated neutron source (or random) pulse train and an ideal detector head provided a convenient means to compare simulated data to probability theory. An additional check that the data was behaving as expected for the paralyzable model was to calculate the expected turning points of the graphs in figure 1. The turning point of the graphs should appear at $\frac{1}{\delta}$.

The data shown in the curves in figure 1 were fit to the form of the paralyzable model given by equation 1. The deadtime parameter was the input parameter in the curve fitting in each case. The χ^2 distribution was applied to this curve fitting [10]. The number of degrees of freedom could then be calculated by the number of data points minus the fit parameter, the input deadtime parameter. The deadtime corrected rates used in each case were assumed to be known i.e. those rates measured when the deadtime parameter was set to zero. Calculated χ^2 values were found to be approximately equal to the number of degrees of freedom in each case. These data are therefore shown to be in good agreement with the ideal paralyzable model of deadtime to within the statistical limits of the simulations. In principle the statistical precision can be reduced to arbitrary levels by generating longer pulse trains. In practice simulation times and memory requirements set practical limits.

Calculation of deadtime correction factor (CF) for uncorrelated neutron source

The deadtime correction factor (CF) was calculated for the random pulse trains at each of the deadtime values using the measured rates and deadtime corrected rates i.e. measured rates at $\delta = 0$, without simulation of the effects of deadtime, are equivalent to the deadtime corrected rates. The deadtime CF is applied to the measured rate in order to derive the deadtime corrected Singles rate as in the following two expressions:

$$S_c \approx S_m \exp(-\delta \cdot S_m) \qquad \text{(Eq. 2)}$$

$$S_c \approx S_m \cdot CF \qquad \text{(Eq. 3)}$$

Re-arrangement of equations 2 and 3 gives the traditional 'first order' exponential form of the CF:

$$CF \approx \frac{S_c}{S_m} \approx \exp(-\delta \cdot S_m) \qquad \text{(Eq. 4)}$$

Experimentally, S_c is unknown and equation 1 is used to estimate S_c from S_m given the value of δ. Since there is no direct method of solution of equation 1, hence the true rate cannot be solved explicitly; instead when equation 1 is used the deadtime correction factor is evaluated by nested exponentials up to the limit of seven. This amounts to finding the solution numerically by iteration and since the solution converges rapidly seven cycles is sufficient for our needs.

Equation (4) is only an approximation to equation 1 in the low rate limit. A convenient higher order approximation adequate in many applications is given by the following relation:

$$CF \approx \exp(-\frac{a + b \cdot S_m}{4} \cdot S_m) \qquad \text{(Eq. 5)}$$

Note, with $b = 0$ and $\frac{a}{4} = \delta$ this collapses to low rate approximation which applies when the deadtime is low (e.g. many amplifiers are in the system and the system is not being taxed so that b may be set to zero). The factor of 4 in equation 5 has been included solely to conform to a convention commonly adopted in this specialism.

In simulation, however, both S_c and δ are known as input parameters and S_m is known as an output parameter, hence the deadtime correction factor in the form of equation 1, the full form evaluation according to the nested exponential technique, may be checked. Re-arrangement of equation 1 gives the following ratio:

$$\zeta_S = \frac{S_c \cdot e^{-\delta \cdot S_c}}{S_m} \qquad \text{(Eq. 6)}$$

For a purely random source, the ratio ξ_S should be unity. This ratio is plotted in figure 2 showing ξ_S against S_c. This again shows that the simulations perform correctly within the sampling uncertainty. The advantage of presenting the data in this way is that the uncertainty estimates are now clearly visible.

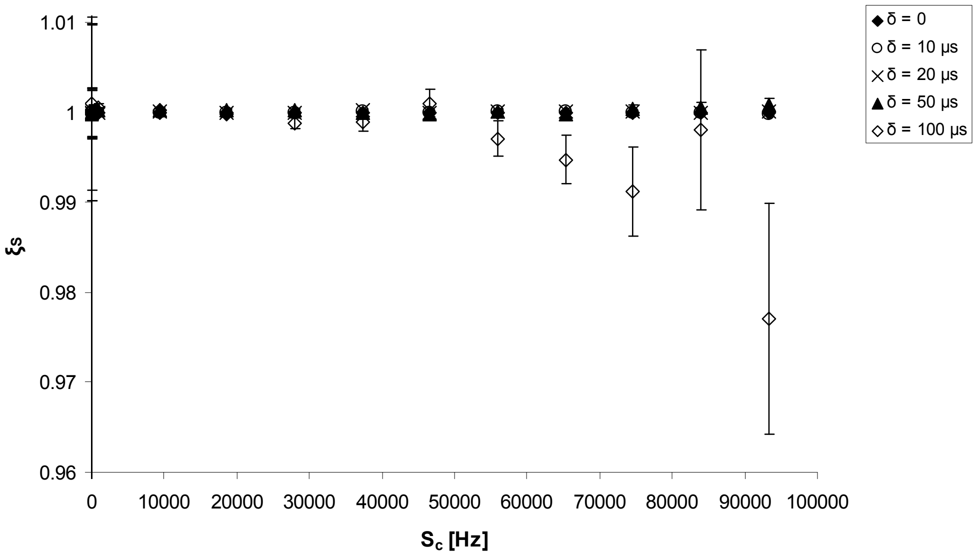

Fig. 2. Ratio ξ_S against deadtime corrected Singles Rate, S_c for an uncorrelated neutron pulse train with paralyzable deadtime

From observation of the data in figure 2 it can be seen that the greatest deviation of the ratio ξ_S from unity occurs at a deadtime value of 100 μs. This does however represent a rather extreme value of deadtime for the present discussion. Statistical uncertainties in the ratio ξ_S are derived from the error in the measured Singles rate. In the case of δ=100μs the precision is also poorest so the deviations are not especially significant. We should also point out that all the plots are correlated in a subtle way because they are derived from the same master pulse train.

Statistical analysis of the measured Singles rates (uncertainty bands reported at the ±1σ level) were based on segmented pulse train analysis to enable estimation of the dispersion in the mean rates, based on replicate counting of an assay item. The 1000 s pulse train was divided into shorter segments representing 20 assay cycles, each of 50 s in duration. Uncertainties are expressed as the standard error in each of the mean rates i.e. representing the best estimate of the standard deviation, σ, on the train in total (not the standard deviation on an individual 1/20th segment).

Once it has been validated that the correct deadtime behaviour has been applied to the pulse train data for a random (uncorrelated) source, the analysis tool can be extended to investigate the effect of correlation on the applied deadtime correction factors.

Calculation of deadtime CF for correlated neutron source

The traditional deadtime correction formulism for NCC Doubles counting relates the deadtime corrected Doubles count rate, D_c to the measured Doubles count rate, D_m through the following expression:

$$D_c \approx D_m \exp(-4 \cdot \delta \cdot S_m) \qquad \text{(Eq. 7)}$$

In practice, equation 7 is again only an approximation applicable at modest rates. The Doubles CF would therefore normally be applied, by analogy to equation 5 as follows:

$$CF_D \approx \exp(-(a + b \cdot S_m) \cdot S_m) \qquad \text{(Eq. 8)}$$

The current work may be extended to investigate the theoretical basis and empirical justification of the factor 4 in the deadtime CF for Doubles i.e. that it may be taken as the Singles CF raised to the fourth power. Correction factors for both uncorrelated and correlated neutron sources may then be directly compared.

CONCLUSIONS AND FUTURE WORK

In summary, the simulation data is behaving as expected for a random pulse train when compared to the paralyzable model of deadtime. Modifications have been made to the simulation code in comparison to earlier versions in published work [3] [4], such as the extension to higher count rates and the addition of overlaying deadtime effects at the pulse train generation stage. This work can be extended to investigate correction factors for correlated trains which will be addressed in future work. In particular, actual items in specific assay systems can be mimicked and the deadtime modelling can be refined to include action of the discriminator and pulse handling circuitry.

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