A New Neutron Multiplicity Deadtime Scheme - 8383

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ABSTRACT

Deadtime corrections for passive neutron coincidence counting are traditionally formulated in terms of the Totals counting rate. The deadtime correction is exponential in form with the effective deadtime being linear in terms of observed Totals rate. The deadtime coefficient for the Reals rate is traditionally fixed at four times that of the Totals rate parameter. When it comes to multiplicity counting, however, more complex expressions are typically used for the Doubles and Triples rates based on mathematical actions to the multiplicity histograms with the Singles (or Trigger) rate being treated rather simplistically. Since the Totals & Singles and Reals & Doubles, respectively, are effectively equivalent measures, the difference in deadtime treatment results is an inconsistency. Furthermore, additional empirical correction factors are often applied in the case of the multiplicity deadtime corrections and these do not follow from the underlying theoretical framework.

The purpose of this paper is to re-examine the semi-empirical deadtime correction expressions from a fresh perspective. We propose to a scheme whereby Totals and Singles are treated equivalently with the correction having the transcendental form of the paralyzable model. The impact of correlations on the Totals deadtime correction is shown to be modest. The deadtime correction factor for Reals and Doubles are again treated similarly also using an exponential form in terms of the corrected Total event rate but with a deadtime parameter which is not fixed ahead of time to be four times that used in the Totals correction. In the case of the Triples correction, which is evaluated from a composite expression, the deadtime corrections for the Singles and Doubles are used as appropriate but a new empirical correction, again given in terms of the corrected rate, is introduced. The new correction acts only on the part of the Triples expression which is does not represent the correlated-accidentals.

The new scheme is not based on an elaborate mathematical model for deadtime losses and consequently does not involve a deadtime-function-weighted sum of histogram elements. In that sense it is simpler to understand and implement. Determination of the free deadtime parameters is based on first extracting the deadtime parameter for the Singles for which several options are available. The Doubles deadtime parameter is then based on rendering the corrected Doubles to Singles ratio for Cf-252 invariant to trigger rate. Similarly the Triple deadtime parameter is extracted from Cf-252 data given the Singles (or Trigger) and Doubles parameters.

In this article we briefly review current practice, lay out the new concept and illustrate their application with experimental data of a passive neutron multiplicity counter.

INTRODUCTION

Both Passive Neutron Coincidence Counting (PNCC) and Passive Neutron Multiplicity Counting (PNMC) techniques are important non destructive assay methods used in the quantification of plutonium and other spontaneously fissile materials across the nuclear fuel cycle. In these techniques, the potential accuracy limiting factor, besides statistical precision, is the dead-time (rate loss) effects.

As the detector efficiencies have increased and the multiplicity ('Triples') technique has been applied to greater Pu masses with higher self-multiplication and also higher random-to spontaneous fission neutron ratio there has been a corresponding drive to reduce the significance of dead time (DT). This has historically been achieved by:

- Designing detector assemblies with a shorter die-away time so that they may be operated effectively with correspondingly shorter coincidence gate widths.
- Distributing the counting efficiency between many and faster preamplifier/discriminator units; ultimately each ^{3}He proportional counter can be serviced by a dedicated module.
- Marshalling the logic pulses of minimal width through derandomizer/summing circuitry into fast multiplicity register electronics.

These methods have been extremely successful and the state of the practice is likely to be extended even further in the near future, especially for special needs. However dead-time corrections remain necessary and improved dead-time treatments therefore offer another option for reducing the assay uncertainty.

Presently, the NCC and NMC techniques employ slightly different dead-time correction schemes. This leads to possible inconsistencies in the measurements of the dead-time corrected Totals and Singles, and Reals and Doubles. It became evident that a unification approach is needed between the NCC and NMC dead-time correction methods. Moreover, a simplification of the NMC higher order multiplets (Doubles and Triples rates) dead-time correction that is mathematically elegant and self consistent is desirable.

CONVENTIONAL NEUTRON COINCIDENCE COUNTING DEADTIME CORRECTION

In conventional Neutron Coincidence Counting (NCC), the observed (or measured) Totals and Reals counting rates, T_m and R_m respectively, maybe be corrected for dead-time losses to yield estimates of T_c and R_c for the corresponding true (or correct) rates using different approaches. A common approach which has been established for many years [1, 1] is given by equations (1) and (2).

$$T_c = T_m \cdot e^{(a+b\cdot T_m)\cdot T_m \cdot \frac{1}{4}} \qquad \text{(Eq. 1)}$$

$$R_c = R_m \cdot e^{(a+b\cdot T_m)\cdot T_m} \qquad \text{(Eq. 2)}$$

where *a* and *b* are empirical parameters to be determined for the system usually based on preserving the R_c/T_c ratio for a defined fissioning system (such as may be achieved using a ^{252}Cf spontaneous fission source) over a suitable trigger rate or dynamic range. According to these expressions the relationship between the Totals and Reals dead-time correction factor is defined by the model and so optimizing on the R_c/T_c ratio is sufficient to extract *a* and *b*.

An alternative approximation [2] takes the form given in equations (3) and (4):

$$T_c = T_m \cdot e^{a \cdot T_c} \quad \text{(Eq. 3)}$$

$$R_c = R_m \cdot e^{b \cdot T_c} \quad \text{(Eq. 4)}$$

where *a* and *b* are empirical parameters pertinent to this scheme which again have to be determined for the system in question. In this approach, it is expected that the ratio *b*/*a* is approximately equal to 4 so that the Reals correction factor is close to being the Totals correction factor raised to the fourth power but this behavior is not imposed by the model nor is it forced during the analysis. The dead-time correction parameters can be determined experimentally by several methods following one of these approaches:

- Using the twin source method [2]. This involves i) measuring the background rates; ii) measuring the rates with source 1 in the counter alongside an inactive dummy source, iii) measuring the rates with both sources 1 and 2 present with source 2 occupying the position previously held by the dummy source, and iv) measuring the rates with source 2 present but source 1 replaced by the dummy source. The values of the dead-time coefficients are then calculated by solving equations (5) and (6). The subscripts "1", "2", "12", and "b" refer to the rates measured with source 1, source 2, sources 1 and 2 both present, and background rates respectively. First the Totals rate is treated and then the Reals rate is treated using the estimated true Totals rate from the first step.

$$T_m^{12} \cdot e^{a \cdot T_c^{12}} + T_m^{b} \cdot e^{a \cdot T_c^{b}} - T_m^{1} \cdot e^{a \cdot T_c^{1}} - T_m^{2} \cdot e^{a \cdot T_c^{1}} = 0 \quad \text{(Eq. 5)}$$

$$R_m^{12} \cdot e^{b \cdot T_c^{12}} + R_m^{b} \cdot e^{b \cdot T_c^{b}} - R_m^{1} \cdot e^{b \cdot T_c^{1}} - R_m^{2} \cdot e^{b \cdot T_c^{1}} = 0 \quad \text{(Eq. 6)}$$

- Using a series of Cf-252 sources to vary the trigger rate. The dead-time coefficients are then extracted by requiring the invariability of the dead-time corrected Reals-to-Totals ratio in the chosen trigger rate domain [4].

- Keeping a fixed correlated source while altering the random trigger rate by introducing an (α, n) emitter such as Am/Li sources. The dead-time corrected Reals rate is invariant for the dynamic range in question. This method was studied in details in separate contribution to this conference [5].

Note that in both schemes the Reals correction factor depends only on the Totals rates and not on the correlated rate from the source. At modest rates both correction approaches work comparably well and both approximate the Totals correction according to either of the two usual ideal types of dead-time, the paralysable or non-paralysable models [6].

However these models are only approximations to actual systems and more over strictly only apply to random (Poisson) pulse trains. As the proportion of correlated to random events increases (as it may when the detection efficiency is increased) one might expect these forms to become less accurate. To appreciate this one can imagine there being a higher probability of short inter-pulse separations in a correlated pulse train and hence a higher chance for dead-time losses. In this case, the Reals correction factor should therefore depend on the histogram distribution which is item and assay dependent. In reality the true Totals and Reals rates are given by the general formula given in equations (7) and (8).

$$T_c = T_m \cdot e^{a_1 \cdot T_c + a_2 \cdot R_c + a_1 \cdot Tr_c + a_4 \cdot Q_c + \cdots} \quad \text{(Eq. 7)}$$

$$R_c = R_m \cdot e^{b_1 \cdot T_c + b_2 \cdot R_c + b_1 \cdot Tr_c + b_4 \cdot Q_c + \cdots} \quad \text{(Eq. 8)}$$

where a_i and b_i are empirical parameters pertinent to this scheme. T, R, Tr, and Q are the Totals, Reals, Triples, and Quads rates respectively. The higher order multiplicities are meant to describe dead-time correction due to correlated neutron events. Because random neutron events (such as the accidental overlap of fission neutrons) dominate the signal pulse train, one can keep only the Totals rates correction in the expressions given in (5) and (6). In other words, a_1T_c>>a_2R_c, a_1T_c>>a_2Tr_c,..etc. Based on work of Srinivasan et al, Ru Haas and others for a counter with an exponential lifetime λ such that d.λ<<1, we estimate: Singles DT correction factor, F_S = {1+[Dc/(f_d.Sc).(δ.λ)] }.exp(δ.Sc). The first term in "{ }" is close to unity for all practical purposes. Therefore, it is easy to evaluate the worst case bias by neglecting it; while the exponential term is simply the 'non-free', full-time, paralyzable deadtime model result.

More importantly, all the previous schemes for dead-time correction assume a functional form for the dead-time correction. Whereas this scheme allows empirical flexibility from the start, recognizing that the real systems are not ideal.

CONVENTIONAL NEUTRON MULTIPLICITY COUNTING DEADTIME CORRECTION

The most wide spread approach to correcting passive neutron multiplicity counter (PNMC) data for dead-time losses is based on that described by Dytlewski [7]. The Doubles and Triples rates are derived from the observed histogram distributions using weighting factors (the so called α and β arrays) based on Vincent's loss factors, which are based on the paralysable (Type II, extendable, cumulative or updating) model. Dytlewski's derivation, however, is not specific about the method to treat the losses in the Singles (or Trigger) rate, which determines how often the coincidence gate is opened. An approximate ad hoc expression, given in equation (9), is used where τ is the dead-time parameter.

$$S_c = e^{\tau \cdot S_m} \cdot S_m \quad \text{(Eq. 9)}$$

Although similar to equation (3), we note the exponent in equation (9) is the DT uncorrected rate. Following the basic method of Dytlewski, the dead-time corrected Doubles and Triples neutron coincidence rates are expressed as follows in equations (10, 11, and 12).

$$D_c = \left\{ \sum_{i=1}^{n} (p_i - q_i) \cdot \alpha_i \right\} \cdot e^{\tau \cdot S_m} \cdot e^{c \cdot S_m} \cdot S_m \quad \text{(Eq. 10)}$$

$$T_c = \left\{ \sum_{i=2}^{n} \beta_i (p_i - q_i) - \sum_{i=1}^{n} \alpha_i (p_i - q_i) \cdot \sum_{i=1}^{n} \alpha_i \cdot q_i \right\} \cdot e^{\tau \cdot S_m} \cdot e^{d \cdot S_m} \cdot S_m \quad \text{(Eq. 11)}$$

$$\alpha_i = 1 + \sum_{j=0}^{i-2} \binom{i-1}{j+1} \frac{(j+1)^j \phi^j}{[1-(j+1)\cdot\phi]^{j+2}}, \; \beta_i = \alpha_i - 1 + \sum_{j=0}^{i-3} \binom{i-1}{j+2} \frac{(j+1)(j+2)^j \phi^j}{[1-(j+2)\cdot\phi]^{j+3}} \quad \text{(Eq. 12)}$$

where S_m is the non-dead-time corrected Singles rate, S_c, D_c, and T_c are the dead-time corrected Singles, Doubles, and Triples rates respectively, $\Phi=\tau/G$, τ is the characteristic dead-time parameter, G is the coincidence gate width, c and d are additional (not present in Dytlewski's original work) empirical Doubles and Triples dead-time parameters to allow extra curvature, and p_i and q_i are the normalized elements of the observed (R+A) and A multiplicity histograms respectively.
Calibration for the dead-time parameters, c, d and τ, is traditionally performed by measurement of a series of ^{252}Cf sources spanning the expected count rate range (typically 1 kHz to 1 MHz). Because there is no significant multiplication or (α, n) reaction rate with the ^{252}Cf sources, the ratios of T/D, T/S and D/S should be constant, once dead-time corrected, independent of source strength. The dead-time correction parameters are then determined adjustment of the parameters to obtain the minimum chi-square value for each of the rates ratios.

Similarly to the conventional NCC dead-time correction, the conventional PNMC dead-time correction outlined above assumes a fixed functional form.

PROPOSED UNIFICATION APPROACH

The actual observed rates (or measured), S_m (Singles), D_m (Doubles), and Tr_m (Triples) respectively, maybe calculated from the Reals-plus-Accidentals (R+A) and Accidentals (A) multiplicity distributions [4]. Thus:

$$S_m = \sum_{i=0}^{\max} A_i \Big/ t_{assay} = N_T / t_{assay} \qquad \text{(Eq. 13)}$$

$$D_m = \sum_{i=1}^{\max} i \cdot [(R+A)_i - A_i] \Big/ t_{assay} \qquad \text{(Eq. 14)}$$

$$Tr_m = \sum_{i=1}^{\max} \frac{i \cdot (i-1)}{2} [(R+A)_i - A_i] \Big/ t_{assay} - S_m \cdot D_m \cdot T_{gate} \qquad \text{(Eq. 15)}$$

where N_T is the total number of events recorded during the counting time t_{assay} and T_{gate} is the gate width. Equations (13, 14, and 15) can be written as:

$$S_m = N_T / t_{assay} \qquad \text{(Eq. 16)}$$

$$D_m = S_m \cdot G_m ; G_m = \sum_{i=1}^{\max} i \cdot [(R+A)_i - A_i] / N_T \qquad \text{(Eq. 17)}$$

$$Tr_m = S_m \cdot H_m - S_m \cdot D_m \cdot T_{gate} ; H_m = \sum_{i=1}^{\max} \frac{i \cdot (i-1)}{2} [(R+A)_i - A_i] \Big/ N_T \qquad \text{(Eq. 18)}$$

We note that the accidental Triples correction term, in equation (18), ($S_m D_m T_{gate}$) may lead to negative values of Tr_m especially for massive (high rate) sources.

In the new dead-time correction scheme we propose, the Totals and Singles rates are treated equivalently with the correction having the transcendental form of the paralyzable model given by equation (9). The impact of correlations on the Totals deadtime correction is shown to be modest.

$$T_c = S_c = S_m \cdot e^{\delta \cdot S_c} \qquad \text{(Eq. 19)}$$

The dead-time coefficient δ should not be thought of as a fundamental physics quantity of the system. It is merely an effective dead-time coefficient that can be assigned to each registered neutron count. It can be experimentally determined using one of the following methods below:

- Using an oscilloscope to measure the smallest time interval between two consecutive pulses that can be detected.
- Placing a near random neutron source, such as may be realized using Am/Li α-n sources, in the neutron assay cavity and record the multiplicity histograms as one would for an assay. The variance to mean-squared is narrower than for a random counting experiment as a result of the dead-time losses. A simple formula exists allowing the dead-time to be extracted from this measurement [8]. This method has been applied and studied for a pair of Passive Scrap Multiplicity Counters (PSMC's). This is the subject of a separate contribution to this conference [5].
- Using the twin source method [2] explained in the previous section.
- This parameter maybe also obtained by imposing the factor of four while correcting the Doubles or Reals rate by using the Cf-252 sources data and requiring the invariability of the dead-time corrected Reals-to-Totals ratio in the chosen trigger rate domain. This method is of particular interest since it lifts the noticeable inconsistency between the neutron coincidence counting (NCC) and multiplicity counting (PNMC) methods concerning the treatment of the dead-time correction.

The dead-time correction factor for the Reals and Doubles rates are again treated similarly (rather than different as they would between PNC and PNMC) also using an exponential form in terms of the corrected Totals or trigger rate but with a Doubles/Reals specific dead-time parameter. Thus:

$$D_c = R_c = D_m \cdot e^{\delta \cdot S_c} \cdot e^{n \cdot \delta \cdot S_c} \,;\; CF_D = e^{\delta \cdot S_c} \cdot e^{n \cdot \delta \cdot S_c} \qquad \text{(Eq. 20)}$$

$$\frac{D_c}{S_c} = \frac{D_m}{S_m} \cdot e^{n \cdot \delta \cdot S_c} \qquad \text{(Eq. 21)}$$

where "n" is the Doubles/Reals dead-time parameter not necessarily fixed ahead of time to be three times that used in the Totals correction. At this stage, there are two methods to handle the Singles and Doubles rates:

- Using a series of Cf-252 sources to vary the trigger rate. The dead-time coefficients are then extracted by requiring the invariability of the dead-time corrected Doubles-to-Singles ratio in the trigger rate dynamic range in question with the multiplicity dead-time parameter determined above during the Singles correction phase.
- The Doubles dead-time parameter can be fixed to three to reach a forced "consensus" between the NCC and PNMC dead-time correction methods.

With the Singles and Doubles rates dead-time correction now properly handled, the Triples dead-time correction is evaluated by rewriting the expression given in equation (18) as:

$$Tr_c = S_c \cdot CF_H \cdot H_m - S_c \cdot D_c \cdot T_{gate}; \frac{H_c}{H_m} = CF_H \quad \text{(Eq. 22)}$$

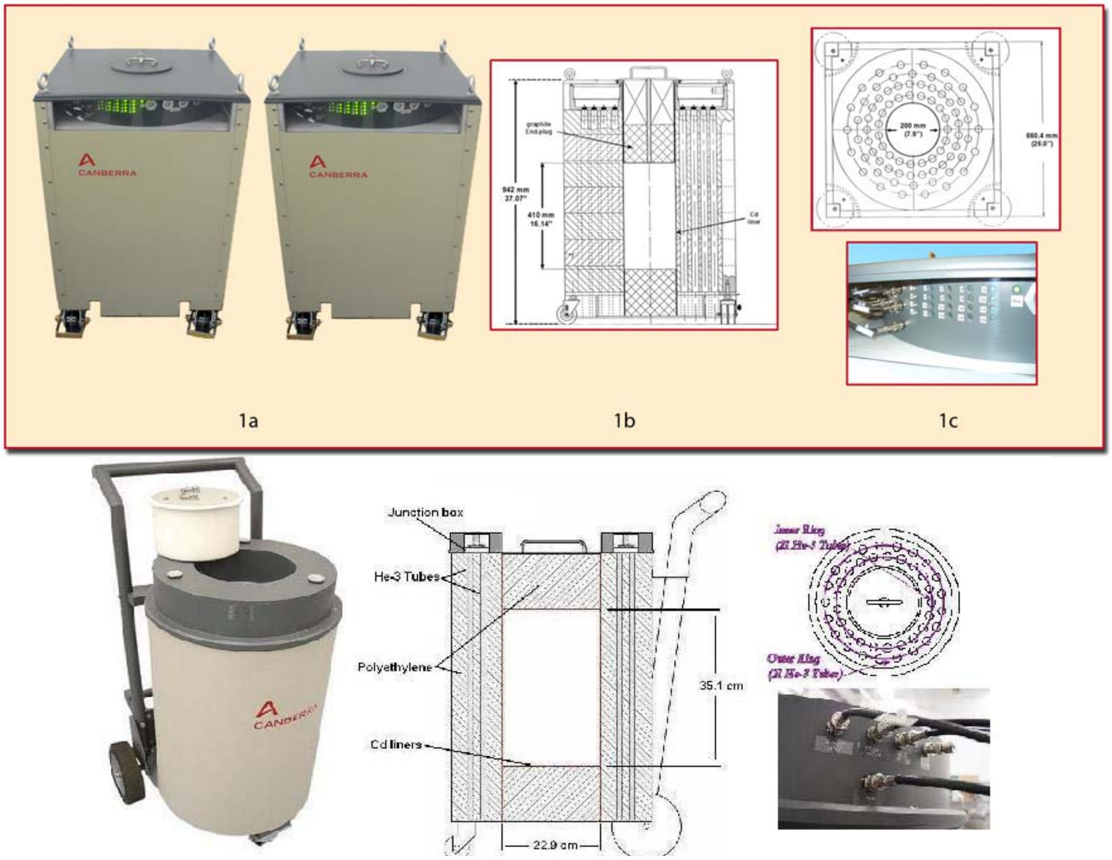
$$\frac{Tr_c}{S_c} = CF_H H_m - D_c \cdot T_{gate} \quad \text{(Eq. 23)}$$

Where we have introduced the new correction factor CF_H applied to the H_m expression. Both the conventional and the proposed NCC and NMC dead-time correction techniques are applied to a Canberra Active Well Coincidence Counter (JCC-51) and a pair of passive scrap multiplicity counters (Canberra model PSMC-01). The results with a comparative discussion are provided in the next section. The objective is to allow the data to reveal to functional behavior of CF_H (vs., for example, S_c).

EXPERIMENTAL RESULTS

The counters used during this study are Passive Scrap Multiplicity Counters (Canberra Model PSMC-01) [9] and a JCC-51 Active Well Coincidence Counter (AWCC) [10]. The PSMC-01 design is based on a multiplicity counter developed by Los Alamos National Laboratory [1]. Figure 1 shows various views of the PSMC-01 counters. The PSMC-01 utilizes 80 ^{3}He proportional tubes arranged in four concentric rings in high-density polyethylene (HDPE). The nominal assay cavity is 200 mm (7.9 inches) in diameter and 400 mm (15.7 inches) tall, lined with 1 mm (0.04 inches) of Cadmium. Graphite end-plugs are used to improve efficiency and make the axial response more uniform. The ^{3}He tubes are divided into 20 individual detector banks via Amptek® JAB-01 boards. The number of tubes per bank varies by ring, in order to even out dead-time, with fewer tubes per bank in the inner most ring. The detector banks are daisy chained through the junction box via a 20 MHz, 16-bits deep de-randomizer board that serves to further reduce the counter dead-time. The main output signal is a 20 ns wide TTL pulse train from all detectors; however, output signals from each ring are also available. The TTL signal is the input for the JSR-14™ neutron multiplicity coincidence analyzer, which is accessed and controlled via the NDA 2000™ software package on a standard PC. The detection efficiency is around 54% and the die-away time is approximately 50 μs. Based on the performed optimization measurements, the PSMC units were operated with a pre-delay of 4.5 μs and a gate-width of 64 μs. The high voltage (HV) setting was 1680 V for one unit and 1660 V for the other one based on Singles, Doubles, and Triples plateau measurements. These values are consistent with the operating parameters used in previous instruments of this type.

The JCC-51, shown in Figure 1, is a high density polyethylene (HDPE) moderated thermal well containing 42 ^{3}He filled proportional counters arranged in two rings. The cavity has an internal cavity of approximately 220 mm diameter and depending on the configuration the cavity can be up to 350mm high. The detection efficiency is around 31% and the die-away time is approximately 52 μs in the 178mm internal height configuration used in this work. Traditionally the shift register electronics are operated with a pre-delay of 4.5μs and a gate-width of 64 μs. In the measurements reported here gate-widths of 32 μs and 128 μs were also used. The HV setting was 1680 V.



2a 2b 2c

Figure 1 (1a) Photograph of the PSMC-01 counters. (1b) Cross section view of the PSMC-01. (1c) Top view arrangement of the concentric rings of ^{3}He tubes, and photograph of multiple signal outputs from the counter. (2a) Photograph of the JCC-51with the end-plug (lid) removed and placed on the junction box. (2b) Cross section view of the JCC-51. (2c) Top view arrangement of the concentric rings of 3He tubes, and photograph of multiple signal outputs from the counter.

For each neutron counter, measurements were made with several sources having neutron emission rates spanning the approximate range 10^3 to 10^6 neutrons per second. To obtain the dead-time parameters for the conventional dead-time correction NCC case a chi-squared minimization is performed on the Reals-to-Totals (Doubles/Singles) ratios obtained from the measurements. Here χ^2 is defined as the difference between the dead-time corrected ratios for each of the individual sources and the average value of these ratios, weighted by the uncertainty in the ratio. The corrected Reals and Totals values (R_c, T_c) are obtained using the expressions given before in equations (1, and 2) with the parameter *a* being varied, while the parameter *b* is set to zero.

In the conventional multiplicity dead-time correction case, it is the Triples-to-Doubles ratio that is used in a similar chi-squared minimization process, with the parameter δ being varied. The parameters "c" and "d" [4] are evaluated simultaneously by requiring that the Doubles-to-Singles and the Triples-to-Singles ratios respectively are constant across all the sources measured.

The resulting dead-time parameters for the conventional NCC and NMC methods are summarized in *Table 1*.

Table 1 Dead-time parameter determination for the JCC-51 and the pair of PSMC-01 using the conventional dead-time correction scheme. The reported uncertainties are of statistical nature.

Parameter	JCC-51		First PSMC-01		Second PSMC-01	
Coincidence Dead time parameters, b=0						
a	812 ns	±4 ns	162 ns	±4 ns	163 ns	± 4 ns
Multiplicity Dead-time Parameters						
δ	207 ns	±5 ns	46 ns	±1 ns	47 ns	±1 ns
c , d	145 ns	±4ns	23 ns	±2 ns	22 ns	±2 ns

Historically, we have found that setting the parameter c = d usually yields the lower reduced χ^2 value. Consequently, this relation was adopted in our conventional NMC dead-time correction scheme presented here.

In the first proposed dead-time (DT) correction method, we unify the NCC and NMC DT correction schemes by constraining the Doubles (or Reals) correction factor to be equal to the Singles (or Totals) correction factor raised to the fourth power (i.e. the Doubles dead-time parameter "n" is fixed to be three in equation (20)). To obtain the dead-time parameter d in equations (19, and 20) a chi-squared minimization is performed on the Reals-to-Totals (Doubles/Singles) ratios obtained from the measurements.

The resulting dead-time corrected Singles and Doubles for the conventional NMC and the proposed unification DT correction methods are summarized in *Table 2*, *Table 3*, and *Table 4*.

Table 2 Comparison of the DT corrected Singles and Doubles rates with both the conventional and proposed techniques using the JCC-51 data. In this proposed technique, the Doubles DT parameter 'n' is fixed to be equal to three.

	DT Corrected Singles [1/s]			DT Corrected Doubles [1/s]		
Source ID	Proposed	Conventional	Deviation	Proposed	Conventional	Deviation
G351	367.37	367.38	0.00%	116.63	116.76	-0.11%
Cf-01-1	16225.56	16231.17	-0.03%	5153.77	5161.75	-0.15%
Cf-04-1	358620.85	359725.69	-0.31%	112917.60	113608.67	-0.61%
Cf-003	14065.13	14069.37	-0.03%	4382.81	4389.17	-0.15%
C6-256	12105.20	12108.35	-0.03%	3833.93	3839.41	-0.14%
95-4	742.44	742.45	0.00%	234.99	235.26	-0.12%
97-13	109614.21	109827.64	-0.19%	34198.54	34314.85	-0.34%

Parameter δ [ns]	**185.41**
Parameter n	**3**

Table 3 Comparison of the DT corrected Singles and Doubles rates with both the conventional and proposed techniques using the first PSMC-01 data. In this proposed technique, the Doubles DT parameter 'n' is fixed to be equal to three.

	DT Corrected Singles [1/s]			DT Corrected Doubles [1/s]		
Source ID	Proposed	Conventional	Deviation	Proposed	Conventional	Deviation
G351	474.06	474.07	0.00%	252.42	252.54	-0.05%
Cf-01-1	20766.79	20772.03	-0.03%	11085.22	11096.11	-0.10%
Cf-04-1	457945.01	460355.69	-0.52%	242380.50	245323.99	-1.20%
Cf-003	18847.22	18851.54	-0.02%	9888.93	9897.81	-0.09%
C6-256	15536.27	15539.20	-0.02%	8300.84	8307.75	-0.08%
95-4	953.08	953.09	0.00%	507.98	508.22	-0.05%
97-13	145610.76	145864.40	-0.17%	76303.15	76619.17	-0.41%

Parameter δ [ns]	**34.04**
Parameter n	**3**

Table 4 Comparison of the DT corrected Singles and Doubles rates with both the conventional and proposed techniques using the second PSMC-01 data. In this proposed technique, the Doubles DT parameter 'n' is fixed to be equal to three.

	DT Corrected Singles [1/s]			DT Corrected Doubles [1/s]		
Source ID	Proposed	Conventional	Deviation	Proposed	Conventional	Deviation
G351	465.68	466.26	-0.12%	248.07	248.19	-0.05%
Cf-01-1	20458.21	20463.78	-0.03%	10931.26	10941.36	-0.09%
Cf-04-1	451292.56	453589.82	-0.51%	239343.53	241911.62	-1.06%
Cf-003	18619.04	18623.76	-0.03%	9783.66	9792.11	-0.09%
C6-256	15328.92	15332.30	-0.02%	8191.50	8198.01	-0.08%
95-4	939.55	940.13	-0.06%	502.11	502.34	-0.05%
97-13	143893.99	144138.04	-0.17%	75459.79	75741.09	-0.37%

Parameter δ [ns]	**34.98**
Parameter n	**3**

It is pertinent to note that the dead-time parameters are not physically and fundamentally inherent to the system in question. They are effective dead-time parameters which depend on the physical system (i.e. the neutron counter and the associated electronics) as well as the correction method through the functional form implementation of the DT parameters. The DT parameters are allowed to differ slightly from one correction method to another but rather we expect the DT corrected Singles, Doubles, and Triples to be the comparable quantities.

At this point of the proposed technique we established both the Singles and Doubles corrections factors to an acceptable extent and unified the treated the Singles and Totals interchangeably; the same is true for the Doubles and Reals. The correction of the Triples is more complicated. The dead-time uncorrected triples can be written in the form given in equation (18). The corrected Triples can be written as shown in equation (22). The main tasks in what follows can be summarized in the determination of the correction factor CF_H as a function of the Singles rate. By taking the ratio of the Triples to the Singles equation (18) can be rewritten under the form in equation (19). Due to the same reason explained above, the Triples-to-Singles ratio is expected to be constant across the measurements. It is estimated by performing an extrapolation of the data shown in Figure 2 for the neutron counters used in this study. The intercept with low or zero Singles establishes the dead-time corrected Triples-to-Singles ratio to a good approximation (to the limits set by the instantaneous rate in the fission bursts). Obviously, this introduces a modest systematic uncertainty to the proposed technique.

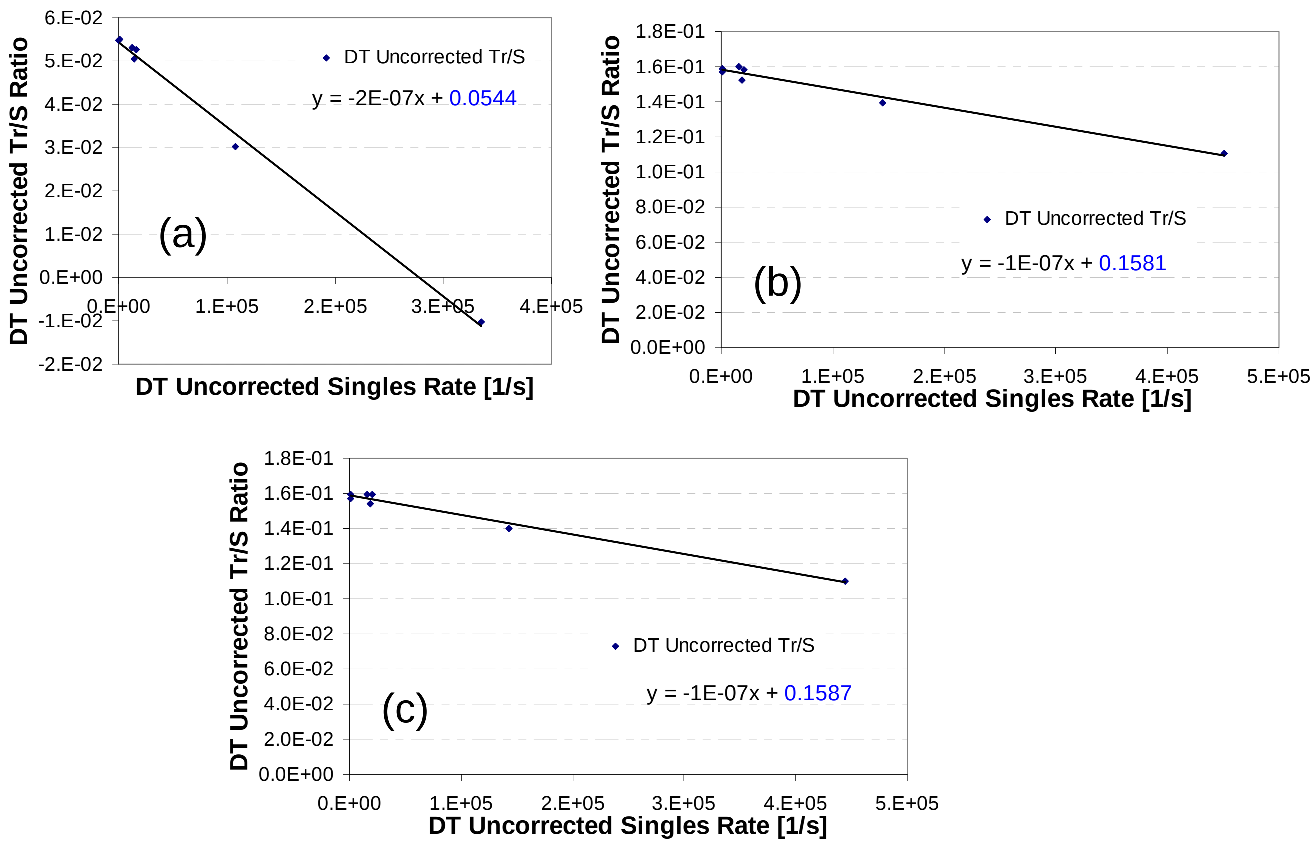


Figure 2 The above plots show the dead-time uncorrected Triples-to-Singles ratio as a function of the dead-time uncorrected Singles. The filled diamonds represent the dead-time uncorrected Triples-to-Singles ratio. The intercept of the linear extrapolation establishes Trm/Sm. The data was acquired with (a) JCC-51 (b) first PSMC-01 (c) second PSMC-01 (64 μs gate width).

The values of the extracted intercepts Tr_c/S_c for the different counters can be injected into equation (23) to establish the functional form of the "H" correction factors CF_H as given in equation (24).

$$CF_H = \frac{Tr_c}{S_c \cdot H_m} + D_c \cdot T_{gate} \qquad \text{(Eq. 24)}$$

where H_m are measurable quantities from the multiplicity histograms as given in equation (18). The measured correction factors are then plotted against the DT corrected Singles rate. The dependence is observed to be linear, as shown in Figure 3, for the three different neutron counters. The functional form is expected to be in the form given in equation (18). Fitting the data determines the Triples dead-time parameter "m". The linear fit provides m close to five and an intercept fairly close to unity. In other words, we are led to:

$$CF_H = (1 + m \cdot \delta \cdot S_c) \qquad \text{(Eq. 25)}$$

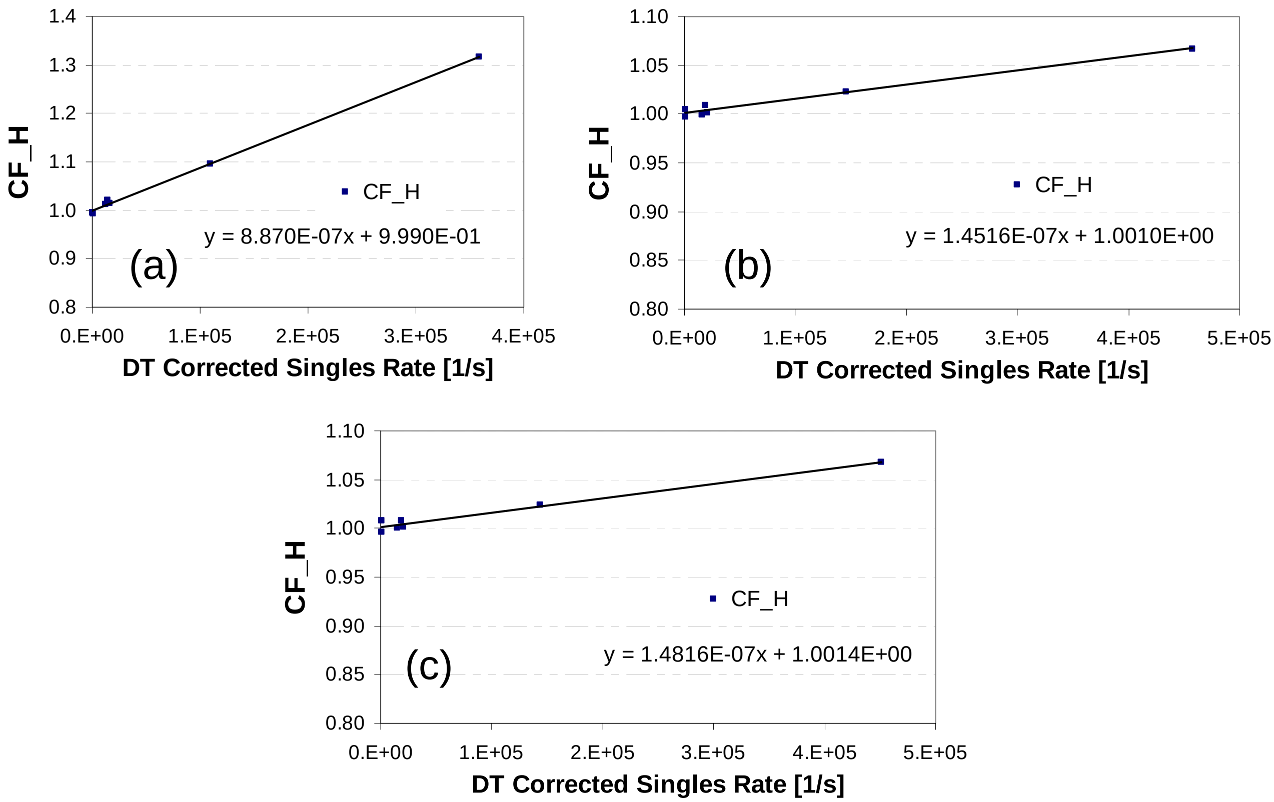


Figure 3 The above plots show the Triples dead-time correction factor as a function of the dead-time corrected Singles. The filled squares represent the data and the continuous line is a linear fit to the data points. The data was acquired with (a) JCC-51 (b) first PSMC-01 (c) second PSMC-01 (64 ms gate width).

The fitting parameters are summarized in **Table 5** and the DT corrected Triples comparison is provided in **Table 6**, **Table 7**, and **Table 8** for the different neutron counters.

Table 5 Summary of the fitting parameters for the different neutron counters.

	JCC-51	First PSMC-01	Second PSMC-01
Parameter "m"	4.8	4.3	4.2
Intercept	0.9990	1.0010	1.0014

Table 6 Comparison of the DT corrected Triples rates with both the conventional and proposed techniques using the JCC-51 data.

	DT Corrected Triples [1/s]		
Source ID	Proposed	Conventional	Deviation
G351	20.26	20.61	-1.71%
Cf-01-1	882.95	927.95	-4.85%
Cf-04-1	19509.25	21162.44	-7.81%
Cf-003	765.42	764.10	0.17%
C6-256	658.80	683.15	-3.56%
95-4	40.67	41.53	-2.08%
97-13	5963.29	6003.48	-0.67%

Table 7 Comparison of the DT corrected Triples rates with both the conventional and proposed techniques using the first PSMC-01 data.

	DT Corrected Triples [1/s]		
Source ID	Proposed	Conventional	Deviation
G351	75.95	76.08	-0.17%
Cf-01-1	3322.23	3354.65	-0.97%
Cf-04-1	74978.26	74460.34	0.70%
Cf-003	2897.95	2919.42	-0.74%
C6-256	2501.11	2518.97	-0.71%
95-4	152.74	152.82	-0.06%
97-13	22219.58	22767.86	-2.41%

Table 8 Comparison of the DT corrected Triples rates with both the conventional and proposed techniques using the second PSMC-01 data.

	DT Corrected Triples [1/s]		
Source ID	Proposed	Conventional	Deviation
G351	74.63	74.68	-0.07%
Cf-01-1	3287.96	3320.65	-0.98%
Cf-04-1	70471.31	73311.57	-3.87%
Cf-003	2890.44	2911.10	-0.71%
C6-256	2464.27	2480.71	-0.66%
95-4	151.37	151.58	-0.14%
97-13	21925.09	22581.47	-2.91%

The proposed new DT correction method outlined has been applied on the same neutron counters by varying the trigger rate (i.e. adding Am/Li sources) to a fixed Cf-252 neutron source. The study leads to the same conclusions outlined in this paper [5].

We note that, this new method can also be applied, using an iterative method, without constraining the Doubles (or Reals) correction factor to be equal to the Singles (or Totals) correction factor raised to the fourth power (i.e. the Doubles dead-time parameter "n" is not fixed to three in equation (20)). In this case, we make use of the fact that the Triples-to-Doubles ratio is invariant across the trigger dynamic range of the performed measurements. Once the Triples DT parameter "m" is extracted from the slope of CF_H as a function of S_c with the Triples-to-Singles ratio trigger invariability, it can be used to extract multiplicity DT parameter δ, as given in equation (25). Then the new parameter δ can be used to derive the Doubles DT parameter "n". Since the Singles and Doubles are DT re-corrected, the Triples rates can also be DT re-corrected with the same proposed scheme. In principle, a small number of iterations will lead to a convergence.

Similarly, if the multiplicity DT parameter was determined using one of the other techniques outlined above, the proposed DT correction scheme can be applied to extract the Doubles and H correction factors CF_D and CF_H respectively.

CONCLUSIONS

Neutron coincidence and multiplicity counting are important non-destructive techniques for nuclear waste characterization and safeguards applications. Dead-time correction is potentially the limiting accuracy factor even at high trigger rate cases. We presented the most common conventional dead-time correction method applied on the Canberra JCC-51 Active Well Coincidence Counter (AWCC) and a pair of Passive Scrap Multiplicity Counters (Canberra Model PSMC-01). Different methods of determining the multiplicity dead-time parameter have been applied so as to shed light onto the problem and a new dead-time correction scheme has been introduced and verified experimentally using these multiplicity counters.

The comparison of the proposed DT correction method and the conventional approach showed excellent agreement while the new proposed method lifted the inconsistency of the NCC and NMC DT correction schemes by unifying the two methods.

Different approaches to apply the proposed DT correction scheme were presented. We hope that this effort will add an enlightened corner to the scientific art of applied coincidence and multiplicity counting techniques.

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