

Dead Time Behaviors in Passive Neutron Correlation Counting

S. Croft, R.D. McElroy, S. Philips and M.F. Villani
Canberra Industries Inc.,
800 Research Parkway, Meriden, CT 06450
USA

L.G. Evans
School of Physics and Astronomy
The University of Birmingham
Edgbaston, Birmingham, B15 2TT
UK

ABSTRACT

Passive Neutron Coincidence Counting (PNCC) and Passive Neutron Multiplicity Counting (PNMC) based on (Multiplicity) Shift Register (M)(SR) pulse train correlation analyzers is a long established and important non destructive assay method used in the quantification of plutonium and other spontaneously fissile materials across the fuel cycle. Very high efficiency neutron chambers (>60%) are now available and are being applied to ever more demanding items including impure materials with a high (α , n) rate and articles with a high self-leakage multiplication. This trend means that high instantaneous count rates are commonly encountered such that the multiplicity histogram extends to high order; in other words the number of events detected in a single coincidence gate can be large. This poses a problem in that the likelihood of accidental (chance) coincidences due to random events and overlapping (super) fission histories increases and precision is lost in correcting for them. The epithermal design is one attempt to reduce the capture time distribution to minimize the accidentals coincidence rate but the field of application is so broad that high instantaneous rates are still encountered. This inevitably results in the need to apply a correction to the observed Singles, Doubles and Triples rate for dead time losses. When the instantaneous counting rate is high the uncertainties in the applied corrections can be the accuracy limiting factor in the derived counting rate. Controlling and compensating for dead time losses so that target accuracy is achieved is a crucial aspect of a successful design and implementation process. Dead time losses can be reduced substantially on new systems intended for special use by using dedicated preamplifier-discriminators for each ^3He -filled proportional counter together with de-randomizer circuitry and fast encoding electronics. These adaptations are costly, however, and may be difficult to retrofit to existing systems. In this work we therefore take a fresh look at the way in which corrections for dead time losses are applied to the recorded MSR data. We note several interesting empirical correlations observed in experimental data which allow dead time parameters to be extracted. We also comment on the self consistency constraints which exist and can be exploited between PNCC and PNMC results and also between expressions for the

correlated rates derived from the random triggered and signal triggered histograms respectively.

INTRODUCTION

At high counting rates the correction of neutron Multiplicity Shift Register (MSR) data for dead-time (rate loss) effects is potentially accuracy limiting compared to the counting precision. In the present context we mean by high count rate when the ‘instantaneous rate’ of a populated histogram channel correspond to a number n -events falling in the coincidence gate of duration T_g , times the dead time per event, δ , gives a product $\phi=(n/T_g)\cdot\delta$ that is a significant fraction of unity.

As the detector efficiencies have increased and the multiplicity (‘Triples’) technique has been applied to greater Pu masses with higher self-multiplication and also higher random-to spontaneous fission neutron ratio there has been a corresponding drive to reduce the significance of dead time. This has been achieved by:

- Designing detector assemblies with a shorter die-away time so that they may be operated effectively with correspondingly shorter coincidence gate widths.
- Distributing the counting efficiency between many and faster preamplifier/discriminator units; ultimately each ^3He proportional counter can be serviced by a dedicated module.
- Marshalling the logic pulses of minimal width through derandomizer/summing circuitry into fast multiplicity register electronics.

These methods have been extremely successful and the state of the practice is likely to be extended in the near future, especially for special needs. However dead-time formalisms remain necessary not least because as the measurement capability is extended new and more demanding challenges emerge and improved dead-time treatments therefore offer another option for reducing the assay uncertainty. In this regard recently some intriguing empirical correlations have been reported. In this work we make some comments on various dead-time correction approaches and present new results that confirm the empirical correlations.

CONVENTIONAL NCC

In conventional Neutron Coincidence Counting (NCC) the observed (or measured) Singles (or Totals or Gross) counting rate S_m and Doubles (pairs or Reals) counting rate D_m are corrected for dead-time losses to yield estimates S_c and D_c for the corresponding true (or correct) rates using one of several empirical approaches. A common approach which has been established for many years [1, 2] takes the form:

$$S_c = S_m \cdot \exp([a+b \cdot S_m] \cdot S_m/4) \quad , \quad D_c = D_m \cdot \exp([a+b \cdot S_m] \cdot S_m)$$

where a and b are empirical parameters to be determined for the system usually based on preserving the D_c/S_c ratio for a defined fissioning system (such as may be achieved using a ^{252}Cf spontaneous fission source) over a suitable dynamic range. According to these expressions the relationship between the Singles and Doubles dead-time correction factor

is defined by the model and so optimizing on the D_c/S_c ratio is sufficient to extract a and b .

An alternative approximation [3] takes the form:

$$S_c = S_m \cdot \exp(a \cdot S_c) \quad , \quad D_c = D_m \cdot \exp(b \cdot S_c)$$

where a and b are empirical parameters pertinent to this scheme which again have to be determined for the system in question. This may be done experimentally by several methods including using the twin source method, using a series of ^{252}Cf sources or by keeping a fixed correlated source while altering the random rate by introducing an (α, n) emitter. When using the twin source method [3] first the Singles rate is treated and then the Doubles rates is treated using the estimated true Singles rate from the first step. This is a powerful technique but as typically applied pegs the correction factors at only two rates when two sources of about equal strength are used (these being the rate of the individual sources and the rate of the two sources combined).

Note that in both schemes the Doubles correction factor depends only on the Singles rates and not on the correlated rate from the source. In the second approach the expectation is that the ratio b/a is approximately equal to 4 so that the Doubles correction factor is close to being the Singles correction factor raised to the fourth power but this behavior is not imposed by the model nor is it forced during the analysis. In practice, for systems operated without a pulse train derandomizer we observe deviations in the ratio b/a from 4 in the range $\pm 10\%$. At modest rates both correction approaches work comparably well and both approximate the Singles correction according to either of the two usual ideal types of dead-time, the paralyzable or non-paralyzable models [4]. However these models are only approximations to actual systems and more over strictly only apply to random (Poisson) pulse trains. As the proportion of correlated to random events increases (as it may when the detection efficiency is increased) one might expect these forms to become less accurate. To appreciate this one can imagine there being a higher probability of short inter pulse separations in a correlated pulse train and hence a higher chance for dead-time losses. The Doubles correction factor should therefore depend on the histogram distribution which is item assay dependent.

PASSIVE NEUTRON MULTIPLICITY COUNTING

The most wide spread approach to correcting Passive Neutron Multiplicity Counter (PNMC) data for dead-time losses is that described by Dytlewski [5]. The Doubles and Triples rates are derived from the observed histogram distributions using weighting factors (the so called α and β arrays) based on Vincent's loss factors which are based on the paralyzable (Type II, extendable, cumulative or updating) model. In principle one could replace these by non-paralyzable factors given by Lang as hinted by Dytlewski however, the paralyzable model is typically the model of choice for multiplicity counters built around gas filled proportional counters. Now, in principle a cylindrical gas filled proportional counter does not have an inherent dead-time, it simply responds to the collection of charge as it was deposited in the gas. However when the analog signal from a charge amplifier is presented to a discriminator/logic pulse generator a processing time (at least equal to the width of the logic pulse) is created for that channel. Events that

have piled-up within the charge collection time of the proportional counter can also be missed by the discriminator because the signal may not fall below the threshold to register the second (or subsequent) events. This is the origin of the paralyzability.

Dytlewski's derivation, however, has nothing to say about how to treat the losses in the Singles (or Trigger) rate which determines how often the coincidence gate is opened. An approximate ad hoc expression, $S_c = S_m \cdot \exp(-\delta \cdot S_m)$, is used, where δ is the dead-time parameter. The Singles and Doubles rates corrected according to Dytlewski therefore do not agree exactly with those estimated using the standard NCC approaches although one would have the expectation that δ would be numerically close to $a/4$ or a respectively for the two NCC dead-time correction forms cited. One could say that this is an indication of the systematic uncertainty in making such corrections but, it is none-the-less a source of inconsistency when NCC and PNMC assays are being compared. To overcome this one could imagine correcting the Singles according to NCC approach and adjusting δ so that the Doubles correction also agrees. With these self-consistency constraints the Triples would now be corrected. In practice this is not done although empirical additional multiplicative factors of the form $\exp(c_k \cdot S_m)$ (or $(1+c_k \cdot S_m)$ to first order - which is essentially equivalent for all practical purposes considered here) are often applied to the three Dytlewski dead-time corrected rates (which we denote by S^* , D^* , T^*), c_k being a free parameter for counting mode k ($k=S, D$ or T with obvious meaning) to be extracted from the characterization data in a best fit sense. These extra correction factors are justified in the sense that the dead-time correction formalism is after all only approximate and is being used only as an empirical guide – for example the assumption that the events for each histogram bin randomly distributed over the duration of the coincidence gate is an approximation for pulse trains that are known to be correlated. In routine use we almost always set c_S to zero in our laboratory in keeping with Dytlewski's suggestion. Optimizations are run allowing c_D and c_T to vary independently but in addition the constraints $c_D=c_T$ and $c_D=c_T/4$ are also applied and the solution which gives the best overall fit to the experimental data is adopted. Independent behavior is in the spirit of treating the additional corrects in a general and empirical way; demanding equality amounts to expecting the additional trigger corrections to be the same for both Doubles and Triples; while the factor of four difference is admittedly arbitrary but seems, based on experience with a number of diverse instruments, to work out well in practice for some systems and is another way of reducing the number of parameters to be extracted by one (from what might be a statistically challenged data set).

It is interesting to note that as commonly tackled the dead-time parameters might be extracted by requiring the ratios (D_c/S_c) and (T_c/D_c) to each be constant over a chosen dynamic range created by ^{252}Cf . In this approach extra trigger correction factors common to all three modes will cancel (part of the logic behind setting c_S to zero). [This problem is explicitly circumvented in the NCC approaches as described earlier.] Of course if we are interested in using the ratio (T_c/D_c) in subsequent calculations this fine detail may not concern us, but if we are concerned with some other combination such as, say $((D_c \cdot T_c/S_c)$, we may expect to get a bias. Thus, as a matter of good practice, it is recommended to optimize on combinations that are sensitive to the parameters of interest and that are of direct interest in application.

Vincent [6] considered the problem of making multiplicity dead-time corrections within the paralyzable model taking into account time correlations but his results do not appear to have been adopted by the community or even independently applied and evaluated which appears to be an omission.

Hage and Cifarelli [7] too developed a detailed mathematical model as to how to correct multiplicity data within the framework of a paralyzable neutron dead-time system with simple exponential die-away profile. This approach also appears to be worthy of more wide spread use subject to more taxing evaluation at higher rates than has hitherto been the case and with state of the current practice detectors. The dead-time corrections for all moments depend on the multiplicity distribution. The treatment, although elegant and masterful is also compact and technical which may partially explain why it has not percolated more widely. In common with all treatments however the approach considers an idealized detector head. The dead-time parameters for a system are typically determined experimentally by loading the detector chains approximately symmetrically. In application deviation from this pattern will emphasize on preamplifier/processing circuit more than another and lead to slightly different losses. It is possibilities such as this that suggest more complete data taking information may prove useful in future endeavors to understand and more completely simulate rate loss effects.

EMPIRICAL CORRELATIONS

In a recent study [8] some interesting and suggestive empirical dead-time relationships have been reported based on the evaluation of some 32 multiplicity systems calibrated using ^{252}Cf as the reference correlated neutron source. It was found in all cases that a plot of (T_m/D_m) vs S_m exhibited, somewhat counter intuitively, a linear behavior. The downward trending line had a slope, λ , related to four times the Dytlewski dead-time parameter, (i.e. $4\cdot\delta$), such that the ratio $(\lambda/4\cdot\delta)$ trended linearly with efficiency. This relationship was confirmed across a wide selection of counters designs and over the entire efficiency range 4.5% to 61% investigated.

We tend to think that plotting (D_m/S_m) vs S_m and (T_m/D_m) vs S_m for a ^{252}Cf source and interpolating to $S_m = 0$ will yield an estimate for these ratios characteristic of the ^{252}Cf spontaneous fission in the detection system in the limit where dead-time effects are negligible. But this is not strictly true. The intercept corresponds to the limiting value when the probability of overlapping fission events tends to zero. Dead-time losses associated with the inherent time correlated multiplicity of neutrons liberated by the fission process will of course still be present. The losses will none-the-less be much smaller than would be the case for a large multiplying item where the fission chain can be extend to far higher multiplicity values and where the probability of fission events overlapping with each other and also with random (α, n) neutrons is high.

An outstanding question left open in our earlier work [8] was how the dead-time parameters and behaviors for a given counter trended with coincidence gate-width. Here we report new measurements taken with a JCC-51 Active Well Coincidence Counter

(AWCC) [10] to settle this issue. We note that according to the standard NCC correct factor approach there should be no impact although within the Dytlewski approach the gate-width appears explicitly in the expressions for the α and β coefficients. For a random pulse train the dependence will drop out since the expected number of events varies in direct proportion to the gate width (i.e. the parameter $\phi=(n/T_g)\cdot\delta$ will stay fixed) *but* for correlated neutron counting shorter intervals are preferred and one might therefore expect a residual effect (especially at extremely high rates in deeply multiplying items).

The JCC-51 is a high density polyethylene (HDPE) moderated thermal well containing 42 ^3He filled proportional counters arranged in two rings. The cavity has an internal cavity of approximately 220mm diameter and depending on the configuration the cavity can be up to 350mm high. The detection efficiency is around 31% and the die-away time is approximately 52 μs in the 178mm internal height configuration used in this work. Traditionally the shift register electronics are operated with a pre-delay of 4.5 μs and a gate-width of 64 μs . In the measurements reported here gate-widths of 32 μs and 128 μs were also used.

For each gate-width setting measurements were made with several sources having neutron emission rates spanning 10^3 to 10^6 neutrons per second. To obtain the dead-time parameters for the NCC case a chi-squared minimization is performed on the Reals-to-Totals (Doubles/Singles) ratios obtained from the measurements. Here X^2 is defined as the difference between the dead-time corrected ratios for each of the individual sources and the average value of these ratios, weighted by the uncertainty in the ratio. The corrected Doubles and Singles values (D_c , S_c) are obtained using the expressions given before with the parameter a being varied, while the parameter b is set to zero.

In the multiplicity case it is the Triples-to-Doubles ratio that is used in a similar chi-squared minimization process, with the parameter δ being varied. The parameters c_D and c_T are evaluated simultaneously by requiring that the Doubles-to-Singles and the Triples-to-Singles ratios respectively are constant across all the sources measured.

For each gate-width three different cases were examined in terms of the relationship between c_D and c_T ; the cases were (i) $c_D = c_T$, (ii) $c_D = 4\cdot c_T$, and (iii) c_D independent of c_T . The resulting dead-time parameters are summarized in the Table I.

Table I. Dead-time parameter determination for the JCC-51 for various gate-width settings.

Parameter	Gate-Width = 32 μs		Gate-Width = 64 μs		Gate-Width = 128 μs	
Coincidence Dead time parameters, $b=0$						
a	894 ns	+/- 3 ns	815 ns	+/- 4 ns	854 ns	+/- 4 ns
Multiplicity Dead-time Parameters						
Case 1: $c_T = c_D$						
δ	215 ns	+/- 5 ns	203 ns	+/- 5 ns	220 ns	+/- 5 ns
c_T, c_D	185 ns	+/- 3 ns	153 ns	+/- 4 ns	187 ns	+/- 4 ns

Case 2: $c_T = 4c_D$						
δ	198 ns	+/- 5 ns	177 ns	+/- 5 ns	194 ns	+/- 5 ns
c_D	224 ns	+/- 3 ns	214 ns	+/- 4 ns	241 ns	+/- 4 ns
c_T	794 ns	+/- 14 ns	857 ns	+/- 15 ns	778 ns	+/- 16 ns
Case 3: c_T independent of c_D						
δ	187 ns	+/- 5 ns	171 ns	+/- 5 ns	157 ns	+/- 5 ns
c_D	251 ns	+/- 3 ns	228 ns	+/- 4 ns	318 ns	+/- 4 ns
c_T	1236 ns	+/- 41 ns	1003 ns	+/- 32 ns	1672 ns	+/- 32 ns

Historically, we have found that setting the parameter $c_D = c_T$ usually yields the lower reduced X^2 value but otherwise have no definitive reason to select this approach for correction of the Doubles and Triples rates. We also note that the values for c_D and c_T will fall into the general range of $0.63 \cdot \delta < c_D = c_T < 0.9 \cdot \delta$. With an average ratio of about 0.76. This information can be used as a starting point in our optimizations and/or as a general check against our experience.

Figure 1 plots the ratios of the non dead-time corrected Triples to Doubles rates versus the non dead-time corrected Singles rates for each of the three gate-width settings. The linear relationship is apparent but the slope of each curve is different. As tentatively suggested in our earlier study [8] the relationship between the slope, λ , of these curves and the dead-time parameter, δ , should have a dependence on the (Doubles) Gate Utilization Factor (GUF). From this data we find empirically that dead-time parameter δ , is inversely proportional to the square-root of the doubles gate utilization factor f_d ($\delta \propto \frac{\lambda}{f_d^{1/2}}$), and recall that the Triples GUF is crudely equal to the square of the Doubles GUF for most counters. The data is summarized in Table II.

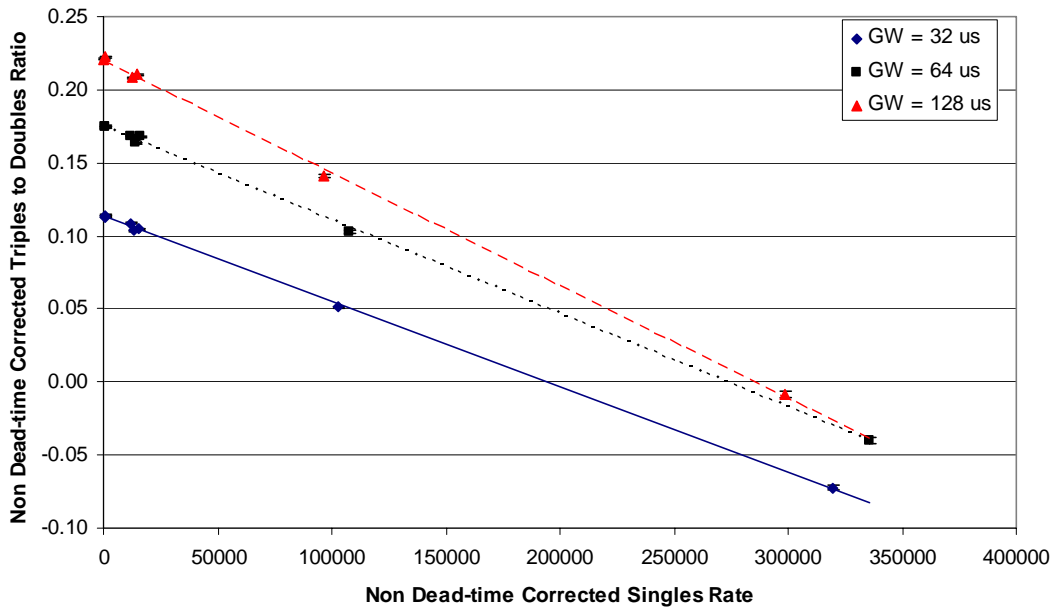


Fig. 1. Plots of the ratio of the Triples to Doubles rates as a function of Singles count rate for three gate width (GW) settings. The same JCC-51 Active Well Coincidence Counter with fixed geometry (fission neutron detection efficiency, $\varepsilon = 30.8\%$) was used for all measurements.

Table II. Comparison of the dead-time parameters determined by the traditional chi-square minimization method and determined from the slope of the Triples to Doubles ratios.

Gate-Width Setting (μs)	Gate Utilization Factor	Dead-Time Parameter (ns)	T/D Slope (λ) (ns)	$\frac{\lambda}{4 \cdot \delta \cdot f_d^{1/2}}$ *
32	0.4191	215.4 ± 5	589 ± 4	1.000 ± 0.014
64	0.6441	202.7 ± 5	650 ± 4	0.999 ± 0.026
128	0.8240	220.2 ± 5	799 ± 4	1.057 ± 0.026

* Note: while the average result for $\lambda/4 \cdot \delta \cdot f_d^{1/2}$ in this table is coincidentally approximately equal to 1, had a different well counter been selected a different average would have been obtained.

We conclude from the data shown in Table II that in addition to a dependence on the neutron detection efficiency, the dead-time has a dependence on the GUF. To demonstrate this statement, Figure 2 plots the correlation between the dead time parameter, δ , and the slope, λ , as a function of detection efficiency for 36 counter configurations without incorporating a dependence on the gate utilization factor while Figure 3 shows the same data with the addition of a scaling factor of $(1/f_d)^{1/2}$. The reduced X_2 for the fit is reduced from 8.27 to 1.03 suggesting this is a valid dependence.

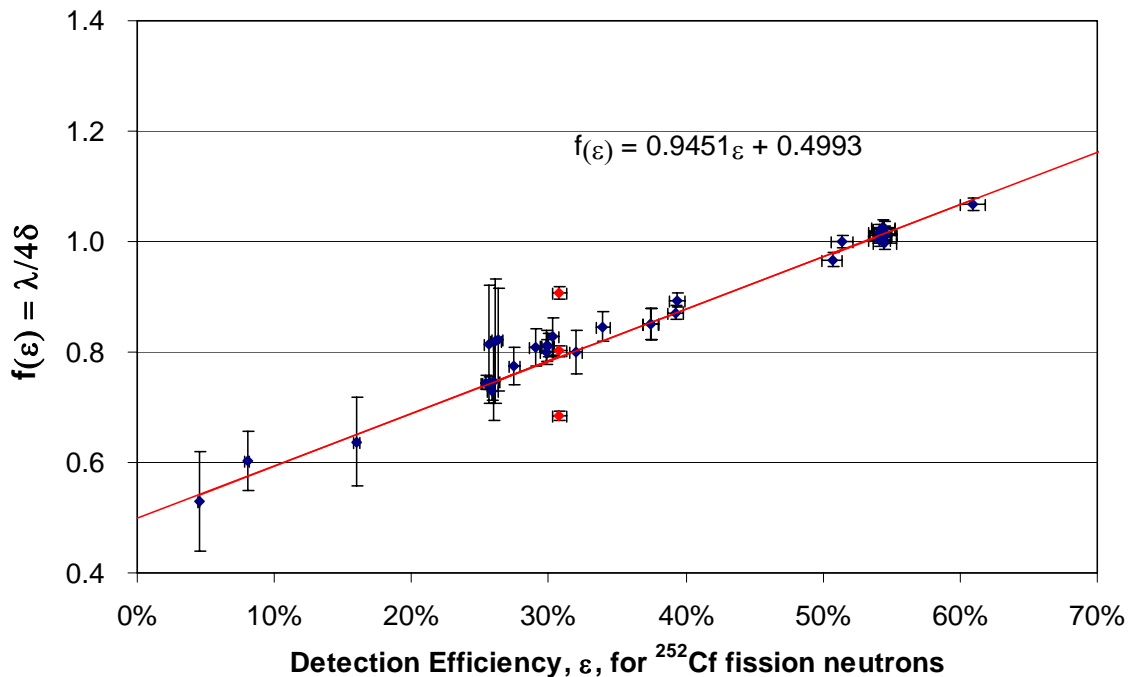


Fig. 2. Observed relationship between the dead-time parameter, δ , and the slope, λ , of the non dead-time corrected Triples to Doubles ratios vs Singles rates, as a function of detector efficiency without adjustment for the GUF.

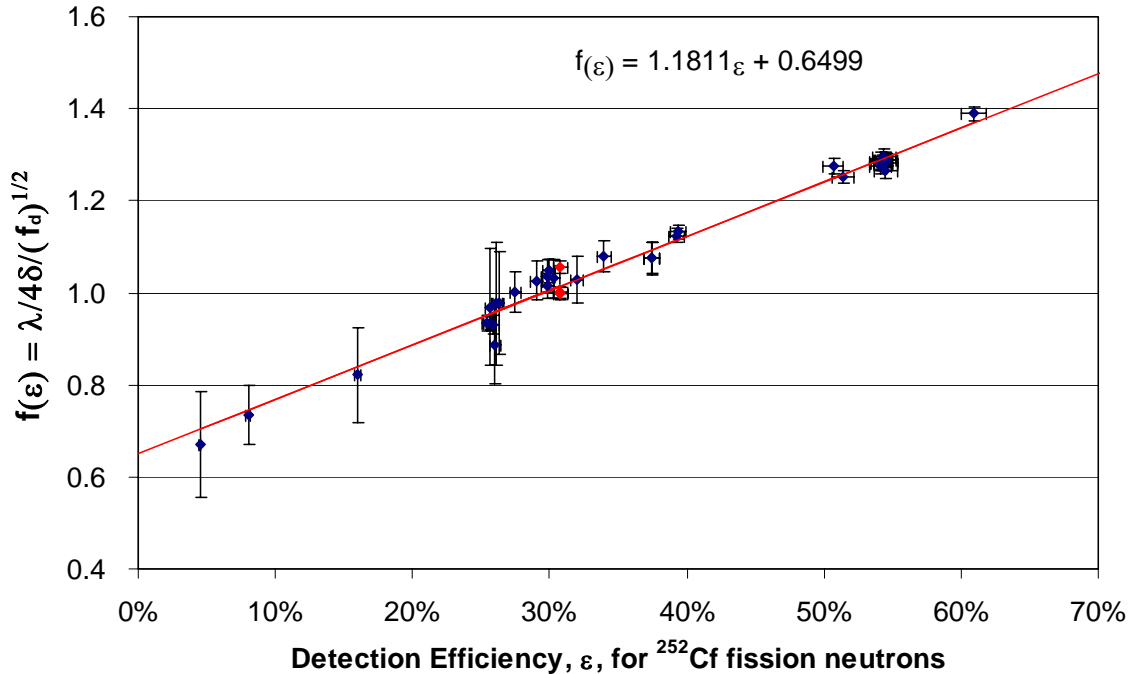


Fig. 3. Observed relationship between the dead-time parameter, δ , and the slope, λ , of the non dead-time corrected Triples to Doubles ratios vs Singles rates, as a function of detector efficiency after correcting for the GUF.

These data serve to show some rather interesting relationships between the dead-time and the basic performance parameters of the coincidence counter and suggest that the dead-time correction models used for multiplicity analysis may not be entirely robust. They also illustrate that to some reasonable level of approximation, suitable for many applications, dead-time parameters can be estimated rather simply without the need for convoluted optimizations.

ON CALCULATING FACTORIAL MOMENTS

In this section we discuss how non dead-time corrected S_m , D_m , T_m and higher factorial moments may be calculated from the Multiplicity Shift Register (MSR) histograms. These are important relationships because they also show the relationships that would exist between dead-time corrected rates and histograms. Recall that each registered event (or trigger) causes the (early) coincident (or Reals-plus-Accidentals, (R+A)) gate to be interrogated and the histogram bin, $N(i)$, corresponding to the number of events, i , inside is increment by one. The (R+A)-gate opens only a short pre-delay period, T_p , after the triggering event and so there is a high chance of events which are correlated in time with it falling within it. We call this the Signal Triggered Inspection (STI) multiplicity histogram. Each event also leads to the (delayed or Accidentals, A) gate to be

interrogated and the histogram bin, $B(i)$, corresponding to the occupancy number, i , is incremented by one. The A-gate is not opened until a long time, T_L , after the (R+A) gate is closed, a time long compared to the characteristic lifetime of neutron in the system, so that all correlations with the triggering event have effectively vanished. Thus, for all practical purposes the opening of the A-gate forms a random sample of the pulse train and for this reason is referred to here as the Random Triggered Interrogation multiplicity histogram (the delayed triggered interrogation histogram would also be suitable and descriptive). The RTI is (statistically) equivalent, apart from for the number of gate openings, to the histogram that would result from a periodic inspection of a gate of equal duration because the pulse train is random in time. In some sense the RTI histogram can be thought of as the background for the STI although it still contains correlated information. The (R+A)- and A-gates are taken to be of equal duration, T_g , commensurate with the characteristic die-away time. In addition we assume here that the number of (R+A)- and A-gates opened are equal as would be the case for a conventional MSR acquisition the action of which has been outlined above. However it is trivial to amend the expressions to the case where the number of openings is different such as may be the case if the A-gate is sampled more frequently. The benefit of this is established in NCC where it is known as ‘fast Accidentals sampling’. In conventional NCC each event opens the delayed A-gate and hence samples a random (in time) segment of pulse train. But an equally good segment could have been chosen a gate width later or sooner. In the limit, in order to improve the precision on how well the A-histogram is known, we could over sample the pulse train by forming a sample every clock cycle of the MSR. In the expressions below we would simply need to apply the appropriate normalization to the action on the histogram.

From the foregoing discussion it may be obvious that the factorial moments can be derived from the STI histogram $N(i)$ alone, the RTI histogram $B(i)$ alone or from a combination of the two. We believe this to be an important technical point in relation to evaluating the rates precisely along with the associated dead-time correction factors although it does not seem to have been looked at in relation to MSR analysis. The basic relations may be derived from a careful and diligent study of the excellent mathematical development given by Cifarelli and Hage [9]. We give expressions not only for the usual Singles, Doubles and Triples but also for Quadruples (Quads) and Pentuples (Pents). In the terminology of Cifarelli and Hage these five rates correspond to the multiplet rates (R_μ/T_M) , $\mu=1$ to 5, multiplied by the corresponding STI GUF [11, 12] which we call f_μ below (f_1 being unity and f_2 being the Doubles GUF etc.)

Before giving expressions for the raw (uncorrected) S, D, T, Q and P rates first let us define how we manipulate the RTI ($B(i)$ or A) multiplicity histogram, the STI ($N(i)$ or (R+A)) multiplicity histogram and the difference (R or $N(i)-B(i)$) histogram:

The difference histogram is defined by:

$$R(i) = N(i) - B(i) \tag{Eq. 1}$$

For the RTI histogram we define the following operations:

$$m_b(1) = \frac{1}{N_T} \cdot \sum_{i=1}^{255} i \cdot B(i) = \left(\frac{t}{N_T} \right) \cdot \frac{1}{t} \sum_{i=1}^{255} i \cdot B(i) \quad (\text{Eq. 2})$$

$$\frac{m_b(1)}{T_g} = \frac{1}{(S \cdot T_g)} \cdot \frac{1}{t} \sum_{i=1}^{255} i \cdot B(i) \quad (\text{Eq. 3})$$

$$\frac{m_b(2)}{T_g} = \frac{1}{(S \cdot T_g)} \cdot \frac{1}{t} \sum_{i=2}^{255} \frac{i \cdot (i-1)}{2} \cdot B(i) \quad (\text{Eq. 4})$$

$$\frac{m_b(3)}{T_g} = \frac{1}{(S \cdot T_g)} \cdot \frac{1}{t} \sum_{i=3}^{255} \frac{i \cdot (i-1) \cdot (i-2)}{6} \cdot B(i) \quad (\text{Eq. 5})$$

$$\frac{m_b(4)}{T_g} = \frac{1}{(S \cdot T_g)} \cdot \frac{1}{t} \sum_{i=4}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3)}{24} \cdot B(i) \quad (\text{Eq. 6})$$

$$\frac{m_b(5)}{T_g} = \frac{1}{(S \cdot T_g)} \cdot \frac{1}{t} \sum_{i=5}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3) \cdot (i-4)}{120} \cdot B(i) \quad (\text{Eq. 7})$$

where T_g is the gate-width (sec) and t is the data acquisition period (sec). N_T is the total number of events recorded which is the same in both the A- and (R+A)-histograms being equal to the number of triggers or inspections. Note the upper limit of the summations is nominally infinity and in practice is the number of the highest histogram populated but we use the value of 255 which is the hardware limit of the MSR electronics in common use in our group.

For the STI histogram we define:

$$S \cdot m_n(0) = S \cdot \frac{1}{N_T} \cdot \sum_{i=1}^{255} 1 \cdot N(i) = \left(\frac{N_T}{t} \right) \cdot \frac{1}{N_T} \sum_{i=0}^{255} N(i) \quad (\text{Eq. 8})$$

$$S \cdot m_n(0) = \frac{1}{t} \sum_{i=0}^{255} N(i) \quad (\text{Eq. 9})$$

$$S \cdot m_n(1) = \frac{1}{t} \sum_{i=1}^{255} i \cdot N(i) \quad (\text{Eq. 10})$$

$$S \cdot m_n(2) = \frac{1}{t} \sum_{i=2}^{255} \frac{i \cdot (i-1)}{2} \cdot N(i) \quad (\text{Eq. 11})$$

$$S \cdot m_n(3) = \frac{1}{t} \sum_{i=3}^{255} \frac{i \cdot (i-1) \cdot (i-2)}{6} \cdot N(i) \quad (\text{Eq. 12})$$

$$S \cdot m_n(4) = \frac{1}{t} \sum_{i=4}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3)}{24} \cdot N(i) \quad (\text{Eq. 13})$$

$$S \cdot m_n(5) = \frac{1}{t} \sum_{i=5}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3) \cdot (i-4)}{120} \cdot N(i) \quad (\text{Eq. 14})$$

For the difference histogram we introduce the following relations:

$$S \cdot [m_n(0) - 0] = \frac{1}{t} \sum_{i=0}^{255} 1 \cdot N(i) \quad (\text{Eq. 15})$$

$$S \cdot [m_n(1) - m_b(1)] = \frac{1}{t} \sum_{i=1}^{255} i \cdot [N(i) - B(i)] \quad (\text{Eq. 16})$$

$$S \cdot [m_n(2) - m_b(2)] = \frac{1}{t} \sum_{i=2}^{255} \frac{i \cdot (i-1)}{2} \cdot [N(i) - B(i)] \quad (\text{Eq. 17})$$

$$S \cdot [m_n(3) - m_b(3)] = \frac{1}{t} \sum_{i=3}^{255} \frac{i \cdot (i-1) \cdot (i-2)}{6} \cdot [N(i) - B(i)] \quad (\text{Eq. 18})$$

$$S \cdot [m_n(4) - m_b(4)] = \frac{1}{t} \sum_{i=4}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3)}{24} \cdot [N(i) - B(i)] \quad (\text{Eq. 19})$$

$$S \cdot [m_n(5) - m_b(5)] = \frac{1}{t} \sum_{i=5}^{255} \frac{i \cdot (i-1) \cdot (i-2) \cdot (i-3) \cdot (i-4)}{120} \cdot [N(i) - B(i)] \quad (\text{Eq. 20})$$

Note that in the Dytlewski scheme of applying dead-time corrections the factors of the form $i \cdot (i-1) \dots (i-(\mu-1)) / \mu!$ which act on the histograms will get replaced by modified expressions that capture the effect of the dead-time [8].

The correlated rates expressed in terms *only* of the RTI histogram are given in equations 21-25. (Note the meaning of the x_μ terms is discussed later, after the *mixed* expressions).

$$S = \frac{1}{x_1} \cdot \frac{m_b(1)}{T_g} = \frac{N_T}{t} \quad (\text{Eq. 21})$$

We note that x_1 is equal to unity and can be omitted.

$$D = \frac{1}{x_2} \cdot \left[\frac{m_b(2)}{T_g} - \frac{1}{2} \cdot S \cdot (S \cdot T_g) \right] \quad (\text{Eq. 22})$$

$$T = \frac{1}{x_3} \cdot \left[\frac{m_b(3)}{T_g} - S \cdot (x_2 \cdot D \cdot T_g) - \frac{1}{6} \cdot S \cdot (S \cdot T_g)^2 \right] \quad (\text{Eq. 23})$$

$$Q = \frac{1}{x_4} \cdot \left[\frac{m_b(4)}{T_g} - S \cdot (x_3 \cdot T \cdot T_g) - \frac{1}{2} \cdot (x_2 \cdot D) \cdot (x_2 \cdot D \cdot T_g) \right. \\ \left. - \frac{1}{2} \cdot S \cdot (S \cdot T_g) \cdot (x_2 \cdot D \cdot T_g) - \frac{1}{24} \cdot S \cdot (S \cdot T_g)^3 \right] \quad (\text{Eq. 24})$$

$$P = \frac{1}{x_5} \cdot \left[\frac{m_b(5)}{T_g} - S \cdot (x_4 \cdot Q \cdot T_g) - (x_2 \cdot D) \cdot (x_3 \cdot T \cdot T_g) \right. \\ \left. - \frac{1}{2} \cdot S \cdot (x_2 \cdot D \cdot T_g)^2 - \frac{1}{2} \cdot S \cdot (S \cdot T_g) \cdot (x_3 \cdot T \cdot T_g) \right. \\ \left. - \frac{1}{6} \cdot S \cdot (S \cdot T_g)^2 \cdot (x_2 \cdot D \cdot T_g) - \frac{1}{120} \cdot S \cdot (S \cdot T_g)^4 \right] \quad (\text{Eq. 25})$$

The correlated rates expressed in terms *only* of the STI histogram are given by equations 26-30:

$$S = S \cdot m_n(0) = \frac{N_T}{t} \quad (\text{Eq. 26})$$

$$D = S \cdot m_n(1) - S \cdot (S \cdot T_g) \quad (\text{Eq. 27})$$

$$T = S \cdot m_n(2) - (1 + x_2) \cdot S \cdot (D \cdot T_g) - \frac{1}{2} \cdot S \cdot (S \cdot T_g)^2 \quad (\text{Eq. 28})$$

$$Q = S \cdot m_n(3) - (1 + x_3) \cdot S \cdot (T \cdot T_g) - x_2 \cdot D \cdot (D \cdot T_g) \\ - \left(\frac{1}{2} + x_2 \right) \cdot S \cdot (S \cdot T_g) \cdot (D \cdot T_g) - \frac{1}{6} \cdot S \cdot (S \cdot T_g)^3 \quad (\text{Eq. 29})$$

$$\begin{aligned}
P = & S \cdot m_n(4) - (1 + x_4) \cdot S \cdot (Q \cdot T_g) - (x_2 + x_3) \cdot D \cdot (T \cdot T_g) \\
& - \left(\frac{1}{2} + x_3 \right) \cdot S \cdot (S \cdot T_g) \cdot (T \cdot T_g) - \left(x_2 + \frac{x_2^2}{2} \right) \cdot S \cdot (D \cdot T_g)^2 \\
& - \left(\frac{1}{6} + \frac{x_2}{2} \right) \cdot S \cdot (S \cdot T_g)^2 \cdot (D \cdot T_g) - \frac{1}{24} \cdot S \cdot (S \cdot T_g)^4
\end{aligned} \tag{Eq. 30}$$

Next we present expressions (equations 31-35) which are *mixed* the sense that they make free use of both the RTI and STI histograms. We have made judicial use of the substitution $m_b(1) = (S \cdot T_g)$ in our mixed RTI and STI expressions.

$$S = S \cdot m_n(0) = \frac{m_b(1)}{T_g} = \frac{N_T}{t} \tag{Eq. 31}$$

$$D = S \cdot [m_n(1) - m_b(1)] = S \cdot m_n(1) - S \cdot (S \cdot T_g) \tag{Eq. 32}$$

$$T = S \cdot [m_n(2) - m_b(2)] - S \cdot (D \cdot T_g) \tag{Eq. 33}$$

$$Q = S \cdot [m_n(3) - m_b(3)] - \frac{m_b(2)}{T_g} \cdot (D \cdot T_g) - S \cdot (T \cdot T_g) \tag{Eq. 34}$$

$$P = S \cdot [m_n(4) - m_b(4)] - \frac{m_b(3)}{T_g} \cdot (D \cdot T_g) - \frac{m_b(2)}{T_g} \cdot (T \cdot T_g) - S \cdot (Q \cdot T_g) \tag{Eq. 35}$$

There is no immediate advantage apparent in making further substitutions if ones intention is to work in terms of mixed RTI and STI histograms. The attraction of the forms given being, in particular, that they do not require the x_{μ} -factors to be estimated. Occasionally however, one does come across in the literature mixed expressions in terms of action on the difference histogram but with the $m_b(\mu)/T_g$ factors in the correlated accidentals correction terms replaced. Thus we arrive at variations for the Q_{MIX} and P_{MIX} rates by substituting:

$$\frac{m_b(2)}{T_g} = x_2 \cdot D + \frac{1}{2} \cdot S \cdot (S \cdot T_g) \tag{Eq. 36}$$

and,

$$\frac{m_b(3)}{T_g} = x_3 \cdot T + x_2 \cdot S \cdot (D \cdot T_g) + \frac{1}{6} \cdot S \cdot (S \cdot T_g)^2 \tag{Eq. 37}$$

Hence we have alternative but formally equivalent forms as follows:

$$Q = S \cdot [m_n(3) - m_b(3)] - \left[x_2 \cdot D + \frac{1}{2} \cdot S \cdot (S \cdot T_g) \right] \cdot (D \cdot T_g) - S \cdot (T \cdot T_g) \quad (\text{Eq. 38})$$

$$P = S \cdot [m_n(4) - m_b(4)] - \left[x_3 \cdot T + x_2 \cdot S \cdot (D \cdot T_g) + \frac{1}{6} \cdot S \cdot (S \cdot T_g)^2 \right] \cdot (D \cdot T_g) - \left[x_2 \cdot D + \frac{1}{2} \cdot S \cdot (S \cdot T_g) \right] \cdot (T \cdot T_g) - S \cdot (Q \cdot T_g) \quad (\text{Eq. 39})$$

These alternative mixed expressions for the Q_{MIX} and P_{MIX} rates may offer some advantage in precision but this remains to be confirmed experimentally using repeat (cycle) data to properly track and quantify the correlated scatter (variances and covariances) in the data.

We are not aware of explicit expressions for Q having previously been given and nor are we aware of the clear demarcation between RTI, STI and mixed expressions being drawn before.

In these expressions the parameter x_μ is the ratio between the RTI and STI gate utilization factors (w_μ/f_μ). A discussion of these can be found elsewhere [13]. Suffice to say that for a useful class of problems they may be taken to be a characteristic of the detector system and determined during characterization and calibration work. The case $\mu=1$ corresponds to Singles for which $w_1=f_1=1$ and hence $x_1=1$. The case $\mu=2$ corresponds to Doubles and so forth. As one might expect, in general for a given counting condition, there is an optimum value of gate-width that minimizes the precision. If the gate is too short correlated events are missed while if it is too long uncorrelated events have a greater chance of being detected by chance in the gate and these chance events must be subtracted – a process that adds uncertainty. The RTI, STI and MIX expressions for the correlated rates are influenced differently by chance events but it turns out that the optimum values w_μ and f_μ are relatively insensitive to gate-width in practice a good compromise can be struck for both simultaneously with a single gate width commensurate with the die-away time of the counter.

We have developed several sets of expressions for the correlated rates. These have been terms the RTI, STI and MIX forms respectively. Within the limitations of counting precision for a particular experiment the rates calculated by each form should be equivalent. In a formal sense counting precision to an arbitrary level can be assumed so that the relationships may be treated as exact identities during characterizations measurements. The implication for dead-time corrections is that an approach developed for one scheme must produce consistent results across all (three) schemes. In other words there are strict relationships and constraints that must exist.

As a case in point the chance pairs rate (“Accidentals”) in NCC may be calculated, as evident above, from the product $S \cdot (S \cdot T_g)$. The corrected Accidentals rate should therefore be consistent with the way in which the Singles rate is corrected. If an empirical approach is being used to make the Singles rate correction then the implication is that the α -coefficients (fixed by the Dytlewski dead-time parameter) should be chosen

to given the same dead-time corrected Accidentals rate – or the argument could be applied the other way about. If the interval density, $f(t)$, of the original process is known it is possible to evaluate the mean rate loss for a single channel for either the extended or non-extended dead-time models [4]. For certain multiplicity counter applications one might expect estimate $f(t)$ reasonable well, iteratively, from knowledge of the spontaneously fissile mass, the (α, n) to (SF, n) ratio and the leakage multiplication of the item. This would be an alternative way to estimate the Singles correction rate independent of the histograms. But self consistency would again be a requirement.

We note that in practice the calculated Accidentals is rarely used in the evaluation of the net Reals rate. This is because although on the face of it the $S \cdot (S \cdot T_g)$ can be determined to a very high statistical precision it is also sensitive to variations in the ambient neutron countrate. The agreement between $A_{\text{meas}} = S \cdot m_{b(1)}$ and $A_{\text{calc}} = S \cdot (S \cdot T_g)$ is, for this reason, often used as a validity check on the measurement and in particular on the constancy of the background during the acquisition. At low rates fluctuations resulting from sporadic cosmic-ray induced neutron bursts come into play and A_{meas} is again often preferred.

It is clear that the higher order correlations depend on the lower order correlated rates. For example three Singles can by chance occur closely in time and mimic a Triple just as a Double and a Single may also by chance combine to mimic a Triple. Thus dead-time corrections established for the lower orders (e.g. the NCC expressions) can be fed into the calculation of the higher order terms.

It is also clear that the various rates are highly correlated through the histograms and the chance events. In applying them in combination the evaluation of the uncertainty from first principles would be a daunting task. For this reason an assay is invariably broken down into a number of shorter counts so that the scatter in the data itself can be used to estimate the variance and covariance in the rates and the relevant combinations directly. A glance at the mixed expressions shows that the Singles rate will always be positive while even at high rates, where due to chance coincidences the $(R+A)$ - and A -histograms become similar, the Doubles rate at least remain consistent with zero subject to statistical fluctuations. However, the expressions also reveal that the higher order rates (triples and above) can be driven systematically negative by chance coincidences at high rates. The challenge of the dead-time correction strategy is to correct for this most effectively and it would seem that the mixed expressions are attractive because they work on the difference histograms.

The NCC dead-time approach applies a correction factor to the observed Singles and Doubles rates to obtain dead-time corrected values. The correction factor for the Doubles is approximately the fourth power of the Singles correction factor. From the form of the equations it is now evident that the use of a simple multiplicative scaling factor will not work for Triples (since the uncorrected rate may be negative which is non-physical). If our understanding of some reports in the literature is correct, however, the expectation at low rates would be that the Triples could be corrected using a factor about equal to the 12th power of the Singles correction factor [14]. In another place, which we do not fully understand yet [15], in the limit of low Singles rate the Doubles and Triples correct

factors would seem to be the square and the cube, respectively, of the Singles correction factor. By the same general argument as invoked above we would not expect an additional multiplicative factor of the form $\exp(c_T \cdot S_m)$ applied to T^* to be appropriate since (e.g. from equation 33) the evaluation of the Triples rate is a composite expression with parts with different dead-time behaviors.

CONCLUSIONS

Multiplicity counting is an established technique in widespread use. The literature is vast and full of pitfalls for the uninitiated. We do not claim to have a complete picture of the field and must apologize to our colleagues if we have omitted a key contribution for the sake of brevity or out of ignorance. We hope that our thoughts can help spur a new wave of interest and exchange on the important topic of dead-time corrections. We have indicated several areas where consistency arguments reveal deficiencies in the currently accepted methods and that may need addressing in the more taxing assay scenarios.

Theoretical dead-time models at best approximate physical reality and will deviate from it to a greater or lesser extent for a particular dynamic range and degree of correlation. Although one would like consistency between various approaches a pragmatic approach would be to apply a plurality of methods and use the differences as an indication of the systematic model dependent uncertainty involved.

We are struck by the outstanding work of a number of investigators working in this field, not least by the efforts at the JRC, Ispra, over two decades ago. The algebraic development of the instrumental point model to the higher multiplets and the theoretical development of dead-time corrections applicable to the multiplicity histograms is tedious and only approximate. This might explain the limited appreciation and relatively narrow uptake of these developments. With the advent of improved transport tools and faster readily available computers we therefore find ourselves drawn to the numerical approach long advocated by Bondár [14]. The point model can be replaced by a detailed Monte Carlo simulation to create a pulse train that can be analyzed in software including allowance for dead-time losses. We can anticipate this approach being used in the near future to explore the kinds of problems addressed in this paper.

Theoretical models serve to guide empirical formalisms and these can be bounded for certain classes of problem. New empirical correlations have recently emerged and we have presented new work with a JCC51 AWCC with respect to dead-time and gate-width dependence. A counter of higher efficiency and operated at higher rates would be useful in looking at the problems highlighted in greater detail.

In our future work we intend to explore a variety of dead-time correction methods, as indicated, and also to evaluate simulation tools which can model the Monte Carlo generated pulse trains from the detector banks and through the various and interconnected elements of the counting electronics. This approach is more flexible than the simple (single paralyzable) analytical models routinely applied today. It allows a distributed system comprising a number of preamplifiers and other components to be considered

with each element having its own type of dead-time behavior. However, it is clear that very long pulse trains and many cases must be simulated to achieve meaningful precision and range and this will in itself present a significant challenge. For investigative purposes experimental studies with list mode data acquisition will be a valuable asset (although data storage and processing times currently preclude such methods for routine assays in the MHz count rate regime which is the target of future systems).

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