Mathematical Modeling to Support Gamma Radiation Angular Distribution Measurements

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ABSTRACT

The development of a functional multidetector device with the capability to precisely locate and characterize multiple gamma sources required sophisticated mathematical modelling incorporating device geometry, material properties and gamma radiation interaction with detector module. This paper describes some aspects of the mathematical modelling performed for the development of ShD-3 multidetector device. Specifically, the methodology for mathematical modelling of gamma-radiation angular distribution measurement for numerous point sources is presented and calibration data confirming adequacy of the model are presented.

INTRODUCTION

Multidetector device ShD (Fig. 1) was successfully employed for gamma-radiation angular distribution measurements at the Chornobyl NPP [1]. Now development of the innovative ShD-3 device is under way. The prototype device, ShD-3, consists primarily of a lead (Pb) sphere containing recessed cadmium-zinc-tellurium (CZT) detectors placed at approximately equidistant intervals around the periphery of the sphere. Crystal size is 6x6x3 mm. Advantages of CZT detectors are small size, high efficiency, relative wide dynamic range, ability to work in high background radiation fields, and some spectrometric capability. The prototype device allows real time data acquisition and control.

MATHEMATICAL MODEL

The mathematical modelling methodology is based on response functions formalism. Here a response function determines counts for each detector under fixed point source position. Every source position has a corresponding set of response functions for multiple detectors that are exposed to the source to a degree dependent upon their location in the device relative to the source. Individual detectors have multiple response functions relevant to multiple sources to which they are exposed.

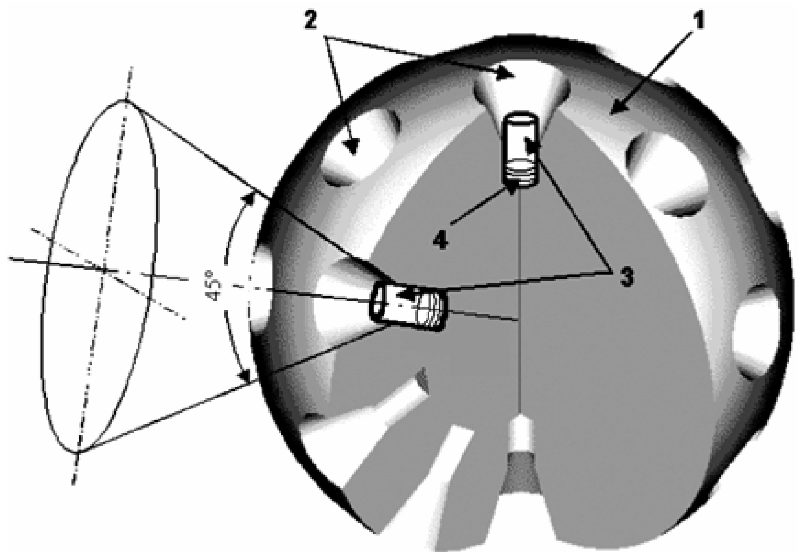

Fig. 1. Geometry of ShD device. 1 – lead body; 2 – collimating hole;
3 – detector capsule; 4 – detector.

Angular Distribution Functions Renewal Procedure

Angular distribution function $H(\theta,\varphi)$ is doze rate observed in the direction defined by spherical coordinates (θ,φ) in the coordinate system with center coinciding with ShD geometrical center. During the measurements all data on the angular distribution is comprised in the detector values.

Renewal procedure exploits the fact that each detector registers some averaged over the solid angle doze rate. This solid angle depends on the collimated hole detector is placed in (see Fig 1., pos. 2). So, approximated distribution function could be written as follows

$$\mathrm{H}(\theta,\varphi) = \sum_{i=1}^{32} \overline{H}_i \; \Theta(\varphi,\theta,\Omega_i)\,, \qquad \text{(Eq. 1)}$$

where $\overline{H}_i$ is doze rate averaged over solid angle Ω, and indicator function $\Theta(\varphi,\theta,\Omega)$ is defined as

$$\Theta(\varphi,\theta,\Omega) = \begin{cases} 1, (\theta,\varphi) \in \Omega \\ 0, (\theta,\varphi) \notin \Omega \end{cases} \qquad \text{(Eq. 2)}$$

Averaged values $\overline{H}_i$ are directly related to response functions. According to definition response function give Sh-D device detectors values for point source with fixed position. In fact, they constitute vector-function for the source angular coordinates:

$$\mathbf{F_i} = \mathbf{F_i}(\theta_i,\varphi_i) \equiv (H_1^{\det},\ldots,H_{32}^{\det})\,, \qquad \text{(Eq. 3)}$$

where $H_1^{\det},\ldots,H_{32}^{\det}$ represent doze rate for given point source registered by each of the detectors.

Doze rate values registered by detectors considerably depend on the ShD device geometry and construction. This follows the fact that each detector registers incident gamma-radiation from the collimation hole together with radiation transmitted through the SdD material. As the result values measured by detectors differ from the real values by this additional irradiation values. In order to give account for this additional radiation we can represent measured values $H_i^{\det}$ as

$$H_i^{\det} = \overline{H}_i + \widetilde{H}_i , \qquad \text{(Eq. 4)}$$

where $\widetilde{H}_i$ – doze rate from the transmitted radiation.

Transmitted radiation doze rate $\widetilde{H}_i$ could be approximated by the following expression

$$\widetilde{H}_i = \sum_{\substack{j=1, \\ j\neq i}}^{32} \alpha_{ij} \overline{H}_j , \qquad \text{(Eq. 5)}$$

where α_{ij} are attenuation coefficients for the radiation incident into solid angle Ω_j.

From the (Eq. 1) and (Eq. 2) follows a closed linear system for $\overline{H}_i$

$$\overline{H}_i + \sum_{\substack{j=1, \\ j\neq i}}^{32} \alpha_{ij} \overline{H}_j = H_i^{\det} \qquad \text{(Eq. 6)}$$

Attenuation coefficients were initially calculated from the lead sphere geometry and verified by calibration procedure. On the next stage geometrical model of the complete device ShD-3 using GEANT-3 simulation software was created. Monte-Carlo simulation for this model allowed to study attenuation coefficients dependence on the distance from ShD device and also gave us more precise values of attenuation coefficients.

Simulation results are shown on Figure 2. Here for sake of simplicity transition coefficients are used instead of attenuation coefficients (transition coefficient is an inverse proportion to attenuation coefficient). As it follows from the Figure 2 radiation transition coefficients (and also attenuation coefficients) vary weakly with the distance starting from the 70 cm point. Thus calculated attenuation coefficients and response functions could be employed for the procedure of real gamma-radiation angular distribution renewal for the distant sources.

Point radiation sources modelling procedure

Response functions obtained from the Monte-Carlo simulation provide the way for angular distribution measurement procedure modeling in the case of multiple point radiation sources. Such a procedure requires a considerably large set of response functions for various radiation source positions to achieve a reliable accuracy. Direct calculations for entire spatial angle are too time consuming due to large amount of numerical calculations. We can essentially reduce number of calculations taking into account the symmetry of detection module.

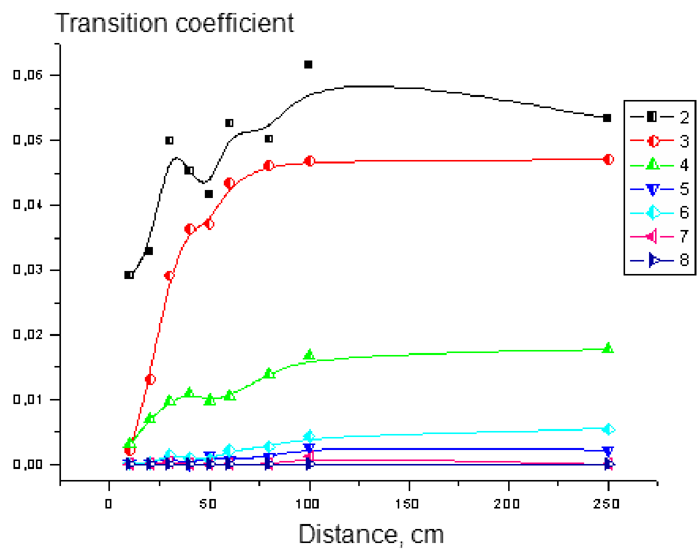

Fig. 2. Transition coefficient dependence on the distance from ShD device for the various detector layers

Symmetry consideration of ShD device shows that detector collimating holes are placed in the centers of icosadodecahedron's faces. So entire detection module symmetry corresponds to icosahedron's spatial symmetry group. Symmetry transformations of that group being applied to response function for given source position result in response function for another source position. So we have to define icosahedron's group presentation in the space of 32-vectors $\mathbf{F_i} = (H_1^{\det}, \ldots, H_{32}^{\det})$

$$\mathbf{F_i'} = \hat{G}\,\mathbf{F_i}\,, \quad \text{(Eq. 7)}$$

where $\hat{G}$ is an operator belonging to icosahedron's symmetry group [2]. Obviously the result of operator $\hat{G}$ action on vector $\mathbf{F_i}$ is some permutation of its elements. Thus operators $\hat{G}$ implement a permutation subgroup. This subgroup is isomorphic to the elements of the crystallographic icosahedron's symmetry group. So, formal procedure requires construction of permutation operators $\hat{G}$ for all elements of icosahedron's symmetry group.

Firstly, it should be noted that symmetry properties of detection module allow us to consider it as icosahedron with collimating holes located in its vertices and centers of its faces. Rotations belonging to the symmetry group obviously lead to transformation of every face into some other side. For the means of permutation operator construction it is sufficient to consider transformations that undergo one fixed icoshedron's face. In our case we shall consider face with detectors 1,7,11 and 2 (see Table 1) as face 1.

Table 1. Face Centers and Corresponding Vertices Numbers.

Face no	Center number	Vertex number
1	2	1, 7, 11
2	3	1, 7, 8

3	4	1, 8, 9
4	5	1, 9, 10
5	6	1, 11, 10
6	12	7, 11, 22
7	13	8, 7, 23
8	14	9, 8, 24
9	15	10, 9, 25
10	16	11, 10, 26
11	17	7, 23, 22
12	18	8, 24, 23
13	19	9, 25, 24
14	20	10, 26, 25
15	21	11, 22, 26
16	27	22, 26, 32
17	28	23, 22, 32
18	29	24, 23, 32
19	30	25, 24, 32
20	31	26, 25, 32

In order to study face transformation one should fix both its location and spatial orientation. For this it is necessary to fix face center (No. 2 in our case) and one of its vertices (No. 1). So every rotation belonging to the symmetry group could be defined by setting the numbers of holes that are holes number 1 and 2 are transformed to. Let these be holes number *p* and *q* correspondingly. Possible values of *p* and *q* are defined by Table 1.

Let $\vec{n}_p$ and $\vec{n}_q$ denote unit vectors pointing from device center to the corresponding holes. Using these vectors it is possible to construct corresponding rotation matrix in Cartesian space. Namely, one could construct three mutually orthogonal vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$, defining the new coordinate system

$$\vec{e}_z = \frac{\vec{n}_p}{|\vec{n}_p|},\qquad \vec{e}_y = \frac{\vec{e}_z \times \vec{n}_q}{|\vec{e}_z \times \vec{n}_q|},\qquad \vec{e}_x = \vec{e}_y \times \vec{e}_z. \tag{Eq. 8}$$

Rotation matrix for this case is given by Cartesian coordinates of vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$

$$G = \begin{pmatrix} e_{x1} & e_{y1} & e_{z1} \\ e_{x2} & e_{y2} & e_{z2} \\ e_{x3} & e_{y3} & e_{z3} \end{pmatrix} \tag{Eq. 9}$$

This matrix defines transformation of detector coordinates to the new coordinate system and is isomorphic to permutation operator $\hat{G}$. Now we can obtain explicit form of the operator $\hat{G}$ by finding correspondence between Cartesian coordinates of detector holes after transformation

$$n'_i = G_{ik} n_k \,, \qquad \text{(Eq. 10)}$$

and initial coordinates. This gives us 32 transformation rules $\{r \to r'; r, r' = 1, \ldots, 32\}$, where *r* is hole number, this way defining permutation operator $\hat{G}$.

So, using the above procedure we were able to obtain response functions for the every face from those calculated initially for the face 1 during the Monte-Carlo simulation. As every response function give detectors counts for the specific point radiation source the set of response functions could be employed for modelling purposes. Namely, one can easily obtain response function for several point sources performing addition of the response functions for each source weighted by the relative intensity coefficient.

The above procedure was implemented using C++ language. We have carried out calculation for two and three sources corresponding to the calibration measurements with real sources (Fig. 3). Perfect agreement achieved proving the correctness of developed modeling method.

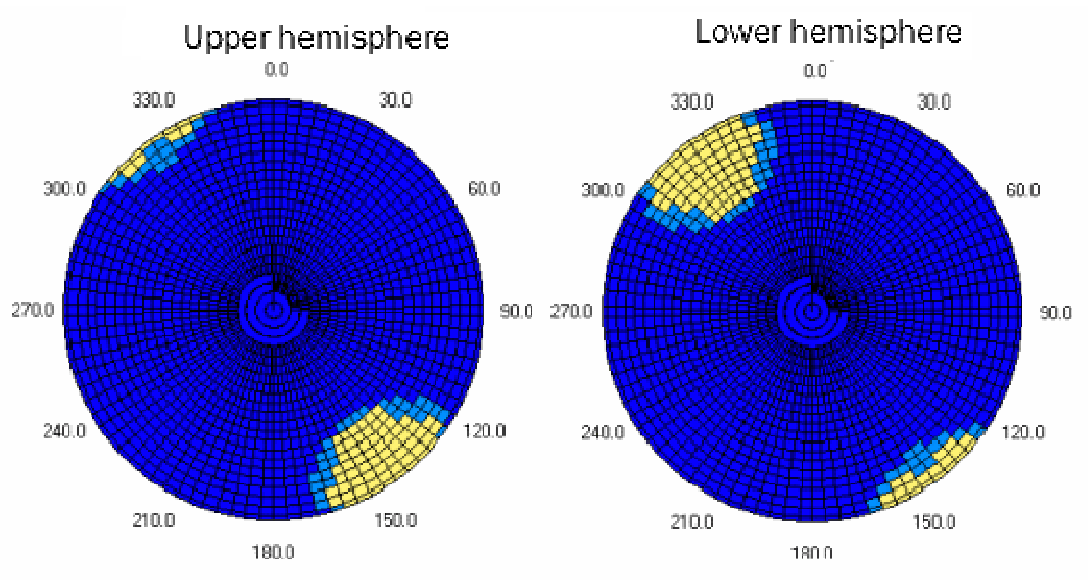

Fig. 3. The result of modeling for two sources.

The developed modelling procedure was also used to study the angular resolution of ShD device. Numerical calculations proved it to be within 20-30 degrees.

IMPORTANCE OF WORK

Multi-detector devices have proven their efficiency in the polluted area of the Chernobyl Nuclear Power Plant “Shelter Object” (SO). But their application is not limited to SO. Radiation accident areas, routine maintenance of nuclear power plants, decontamination and decommissioning of nuclear weapons facilities, and monitoring of radioactive waste storage facilities are places where this mathematical modeling could be successfully employed.

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