

MEPHISTOⁱ : A VOLUMIC MODEL FOR THE CHARACTERIZATION OF WASTE PACKAGES BY GAMMA EMISSION TOMOGRAPHY

Raphaël Thierry, Jean-Luc Pettier,
CEA Cadarache , Laboratoire d'Expertise et de Caractérisation du DCC/DESD/SCCD⁽ⁱⁱ⁾
13108 -St-Paul Lez Durance, France
Phone 04-42-25-61-97, 04-42-25-49-19, Fax 04-42-25-73-77,
E-mail thierry@desdcad.cea.fr, pettier@desdcad.cea.fr

ABSTRACT

We attempt to determine using a small number of measurements, the distribution and the activity of radio-elements contained in radioactive waste packages. A projection simulator for ECT imagery (Emission Computed Tomography) was developed for a parallel-hole collimator placed in front of a HyperPur Germanium detector (HpGe). It takes into account the attenuation, diffusion and the spatial detector response, based on the analytical modelling of the line spread function. The model has been evaluated on packages of homogeneous density using point sources of americium (²⁴¹Am) and europium (¹⁵²Eu), as well as ¹⁵²Eu linear sources. The photonic noise is modeled using a Poisson distribution in which we introduce an *a priori* distribution on the image: we will use a Maximum *a Posteriori* (MAP) estimator for the image restitution. The analysis of the results showed an improved performance compared to other models, both in quantification and in localization of radio-elements. The study underline the importance of a accurate modelling of photonic measurements inherent to the device as well as the influence on quality and stability of the reconstructed images.

INTRODUCTION

Numerous fields of application in nuclear industry produce wastes contaminated in radioactive elements of different nature. The decision on their storage mode in surface or in depth depends on the type of radionuclides they contain (α, β, γ , emitters...) and on their activity. The Laboratoire d'Expertise et de Caractérisation (LEC) of the CEA Cadarache, develops non-destructive methods to qualify the $\beta\gamma$ activity and distribution on Low-Level and Medium-Level (LL & ML) waste packages of several hundred liters. These methods have some advantages over destructive methods :

1. the tomographic examination has no measurement representativity problems : it can be achieved on the entire volume of the package,
2. there is no deterioration of the package, this being an undeniable advantage in terms of safety and radio-protection.

One of the tasks tomographic reconstruction must be able to achieve is the accuracy in quantification in terms of intensity, shape and precision in the localization. This objective is ambitious since the acquisition process is perturbed by many factors. The photoelectric effet causes an attenuation (absorption) through the objet shape that considerably reduces the number of primary photons detectable. Moreover, the deflected photons through Compton scattering, if detected, generate an undesirable noise by adding themselves to the mesure. These effects degrade the localization of the emitting sources and lead to a loss of contrast in the reconstructed images. Moreover, a geometrical loss in focus is at the origine of a spatially varying collimator/detector response. Another source of perturbation is the statistical Poisson-nature of the photonic emission. Compensation for attenuation, detector response ,Compton scattering and emission noise have been the objet of considerable interest

in the field of nuclear medicine over the last twenty years. An abundant literature proposes compensating methods in individual or global ways.

Thus, the attenuation phenomenon through photoelectric effect can be compensated by using an attenuation map via filtering methods (1-5), or through iterative methods (6-9). According to Tsui (7), the latter is more accurate and the most adequate when the attenuation distribution is not constant.

The problem of the Compton diffusion has mostly been treated according to four large families : that working with spectral windows (11), that which attempts to model the phenomenon in the projection/retroprojection operator (12), those based on a weighting of the detected events (13), those attempting to eliminate the scattered through deconvolution methods (14), through spectral analysis (15) or by using a secondary energy window (16).

The effects of spatial response in detection have been corrected using filtering methods on the projections (17,18) by treating projection lines within the detection kernel (19,20), or by accurate modelling of the detection geometry by incorporating it in the PSF⁽ⁱⁱⁱ⁾ (21).

The approaches unifying both the attenuation and Compton diffusion phenomena have been proposed (22, 23) as well as models unifying attenuation and detection response (21, 24). Nevertheless, there is still a need for a method unifying all these phenomena. Moreover, work done on (25) with SPECT (Single Photon ECT) simulators underline the direct dependence between the accuracy of a PSF model and the quality of the reconstruction.

Models unifying the three main phenomena have been proposed i.e. (26, 27) or (28) when the environments are homogeneous in attenuation. A large number of these methods however remain very time and memory consuming, i.e. (26).

In this paper, we propose a method which takes into account the three dimensional variations of detection response and a two dimensional attenuation map (not necessarily homogeneous). The Compton diffusion is taken into account through pre-treatment of the measurement spectra.

However, we must be aware of two major difficulties : a longer calculation time than by projection following the central ray ("Ray-Driven" method) or any other analytical method, as well as noise amplification through the iterations. It is possible to accelerate the reconstruction algorithms using pre-conditioning techniques. The stabilization can be envisaged through the coupling of a smoothing technique with a markovian regularization method made spatially adaptative.

DESCRIPTION OF THE "TEMISEC" ACQUISITION SYSTEM

The TEMISEC (Tomographe d'EMISSION pour l'Expertise des Colis) facility consists of a mechanical part with a self-motoring support moving the package according to 3 axes (θ for rotation, Z for elevation and X for the abscissa). The detection includes a HyperPur Germanium detector of 47 mm in diameter placed in a cylindrical lead shield and with a parallel-hole lead collimator.

The collimator dimension for which simulations and experiments were performed is a 3-cm open square and 38.5-cm long.

The package dimension corresponds to 200-liter waste drums, i.e. a cylinder of 28.5-cm radius, noted R_{obj} and 85-cm high.

The objet attenuation map is obtained either by :

1. a tomodensimetric cross section obtained using a transmission tomograph with an irradiant source of ^{60}Co ,
2. using the CARACO mixtures and compounds method (29), from a global measurement and a reduced Hubbell table (30),
3. using the Klein-Nishina equations and an average density, in the Compton domain for the energy interpolation of global or local linear attenuations.

NOTATIONS AND PRESENTATION OF THE MODEL

We attempt to reconstruct the image f corresponding to the volume activity in a space of finite dimension $N=n^2$,

$$(1) \quad f(x) = \sum_{i,j} f_{ij} \phi_{ij}(x)$$

where the space of base functions f_{ij} corresponds to a classic cartesian discretization, i.e. f_{ij} correspond to the indicating functions of the voxels (also called the kronecker operator). We note $R(O, \vec{i}, \vec{j}, \vec{k})$ the fixed reference object and $(O, \vec{k})$ the rotation axis of the tomographic system. The objet referential is axially positionned at the height of the tomographic examination. The activity f is concentrated in the object domain Ω , of side $\partial\Omega = \{(x,y) / x^2 + y^2 = R_{obj}^2\}$.

We note m the measurement vector, of size $M=n_q n_x$, where n_q is the number of angular positions occupied by the detector and n_x the number of measurements performed by projection.

We will use a vectorial representation rather than a matricial one in order to avoid unusefull overloadind of indexes in equations. The image and measurement indexation is made easier by a lexicographic order : the equivalence between the two indexations is given by :

$$(2) \quad \begin{aligned} \forall i, j \in [1..n]^2, \quad k = (i-1)n + j, & \quad \forall k \in [1..N], \quad i = k / N + 1, j = k \% N \\ \forall \theta \in [1..n_\theta], \forall t \in [1..n_x], \quad l = (\theta-1)n_\theta + t & \Leftrightarrow \forall l \in [1..M], \quad \theta = l / N + 1, t = l \% N \end{aligned}$$

In expression (2), / designates the entire division and % the rest of the division.

The compensation of attenuation, diffusion and of detector response is calculated via the projection matrix $\mathbf{H} \stackrel{def}{=} (\mathbf{H}_{l,k})_{l=1..M, k=1..N}$ of size M -by- N , where element $\mathbf{H}_{l,k}$ expresses the proportion of voxel k to influence on the measurement l . The problem is then classically written under the following matricial form :

$$(3) \quad \mathbf{H}f = m$$

We note by l_{col}, h_{col} , et L_{col} the collimator-hole dimension respectively in sidth, height and length. The voxels height h , will be taken as equal to the exit height of the collimated beam (c.f. Figure 1).

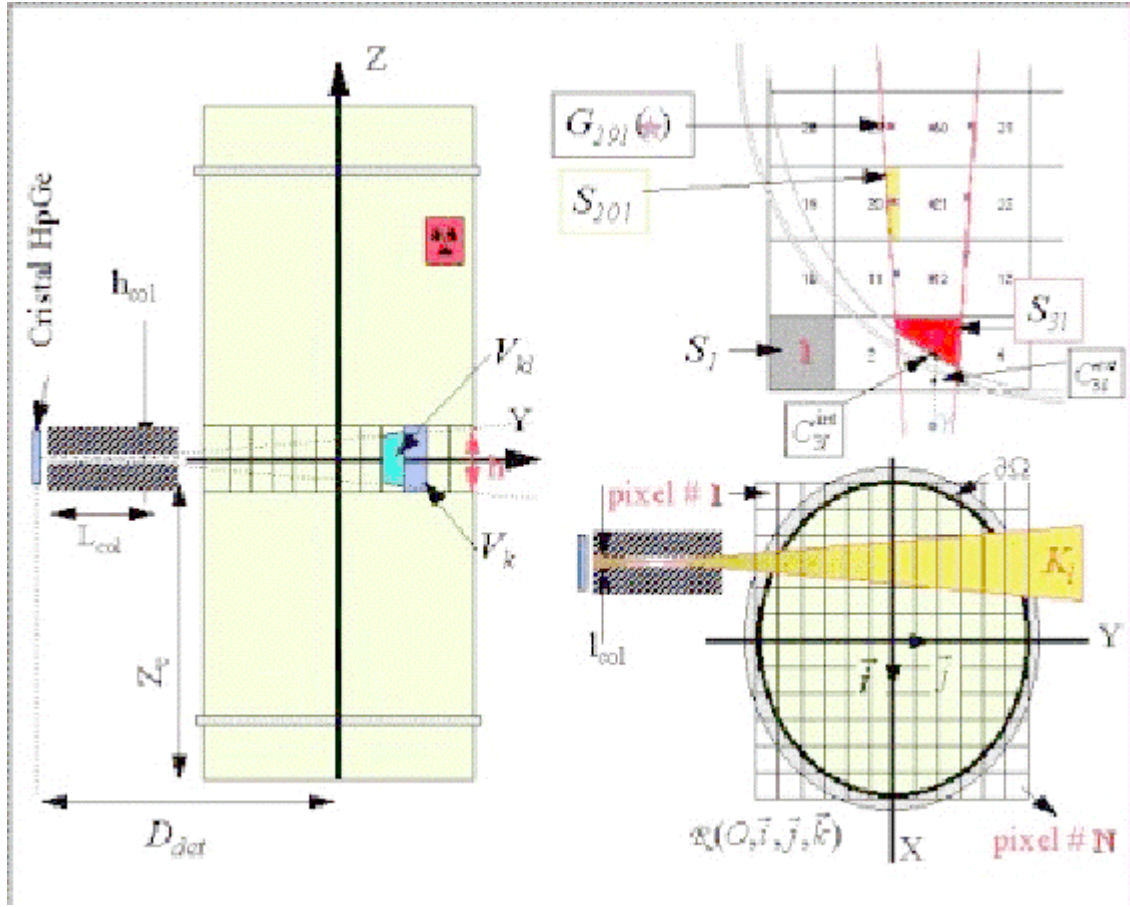


Figure 1: View according to X and Z axes of the acquisition device and illustration of the quantities V_k , V_{kb} , S_k , S_{kl} and K_l

In all the paragraphs below, we will designate by m the linear attenuation function (cm^{-1}) in the cross section plane. In the absence of data from transmission tomography, we will mention as m_{cont} the linear attenuation of the container. The attenuation dependence on the energy is not explicitly expressed for simplicity reasons on the equations writings. Nevertheless we shall remember that for each equation, one must read $m(E)$ and $m_{\text{cont}}(E)$ instead of m and m_{cont} .

COMPENSATION OF THE COMPTON EFFECT

The calculation of the scatter response is based on the evaluation of the Compton continuum to a Galton form (29). We show that this contributing value $N_c(x)$ at energy E_x , for a peak identified at energy E_{pic} , comprised between the energy E_{deb} and E_{fin} , ($E_x \in [E_{\text{deb}}, E_{\text{fin}}]$) is given by the expression:

$$(4) \quad N_c(x) = N_{\text{deb}} + (N_{\text{fin}} - N_{\text{deb}}) \cdot \frac{I(x)}{I} = N_{\text{deb}} + (N_{\text{fin}} - N_{\text{deb}}) \cdot \left(\frac{\sum_{i=E_{\text{deb}}}^{E_x} N(i)}{\sum_{i=E_{\text{deb}}}^{E_{\text{fin}}} N(i)} \right)$$

$I(x)$ represents the partial sum (as an approximation of the partial integral) of the counting rates within the window $[E_{deb}, E_{fin}]$, N_{deb} and N_{fin} are contributing value at energy E_{deb} and E_{fin} ,

I represents the total sum in the energy window identifying the peak, and $N(i)$ the total photon-counts in the energy canal E_i .

The elimination of the scattering component F_{compt} is achieved finally by adding the continuous values thus obtained :

$$(5) \quad F_{compt} = \sum_{x=E_{deb}}^{E_{fin}} N_c(x)$$

The accuracy of the method is greatly guaranteed by the high energy resolution of the Hyperpure Germanium crystal (0.66% at 140 keV). The detector specificity has been used many times to quantify the Compton diffusion fraction in the Anger cameras (31), and has been validated by Monte-Carlo algorithms (32).

COMPENSATION OF ATTENUATION AND DETECTOR RESPONSE

We note M_{det} the detector-central point and $l \in [1..M]$, the index corresponding to the l th measurement. The volume detection kernel, noted K_l , is a square based pyramid with vertex M_{det} .

Let V_k the volume related to the voxel k , and S_k the surface corresponding to the voxel-base on the cross section plane. We note V_{kl} the volume obtained by intersecting V_k with K_l , as well as the surface S_{kl} , intersection of S_k and K_l . These quantities are illustrated in Figure 1. For each measurement, the projection operator of our system is given by :

$$(6) \quad m_l = \iiint_{K_l} f(M).PSF(M)dM = \iiint_{K_l} f(M).R_{det}^p(M).\exp\left(-\int_{M_{det}}^M \mu(l)dl\right)dM$$

where the PSF designates the over-all point response function of attenuation, the source scattering response, the collimator geometric response, septa penetration response, and the intrinsic crystal spreading, and $f(M)$ the volume activity at point M . In our work here, we neglect the crystal scattering because it is relatively insignificant compared with the geometric system response. The second equality of (6) expresses more accurately the decomposition of the PSF as the product of $R_{det}^p(M)$ the spatial response of the system, by the attenuation factor of the environment. We note $\Gamma_l = \{k \in [1..N] / V_{kl} = K_l \cap V_k \neq \emptyset\}$, all the voxels having a non-nil intersection with the detection space. By decomposing the integration space K_l on Γ_l , (6) becomes :

$$(7) \quad m_l = \sum_{k \in \Gamma_l} \iiint_{V_{kl}} f(M)PSF(M)dM = \sum_{k \in \Gamma_l} \iint_{S_{kl}} \left\{ \int_{[-h,h]} f(x,y,z)PSF(x,y,z)dz \right\} dx dy$$

Due to the operation of detection cameras (line integration), it is more convenient to model the linear dispersion function (LSF : *Line Spread Function*), in order to experimentally establish the model. It is obtained by integration following z of the PSF, hence :

$$(8) \quad LSF(x, y) = \int_{z=-\frac{h}{2}}^{\frac{h}{2}} PSF(x, y, z) dz = \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{det}^p(x, y, z) \cdot \exp\left(-\int_{M_{det}}^{M(x, y, z)} \mu(l) dl\right) dz$$

We now admit the first hypothesis $[h_0]$: the attenuation factor is independent from variable z . At this time we can re-write (8) as follows :

$$(9) \quad LSF_0(x, y) = \exp\left(-\int_{M_{det}}^{M(x, y, 0)} \mu(l) dl\right) \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{det}^p(x, y, z) \cdot dz$$

In the case where we have the attenuation map from transmission tomography, we determine the integral calculation by bi-linear interpolation to the pixels close to the attenuation grid along the line connecting the points M and C_{kl}^{ext} by a method similar to the recursive method developed by Joseph (33).

In the absence of tomodensimetric data, we determine C_{kl}^{int} and C_{kl}^{ext} (c.f. Figure 1), the intersections linking M_{det} to M with the inside edge, respectively the outside edge of the drum. The coefficients μ and μ_{cont} are determined, either using Klein-Nishina equations with the apparent density of the object assumed to be physically homogeneous, or by using fabrication data describing the mass composition of the package and its container. The calculation of the attenuation can be simplified to obtain the following expression :

$$(10) \quad \exp\left(-\int_{M_{det}}^M \mu(l) dl\right) = \exp\left[-\mu_{cont} \cdot \|C_{ji}^{ext} - C_{ji}^{int}\| - \mu_{obj} \cdot \|G_{ji} - C_{ji}^{int}\|\right]$$

Hypothesis $[h_0]$ was verified in the framework of homogeneous attenuation and in the case where the attenuation distribution is axially constant : thus, $\forall x, y \quad \mu(x, y, z) = \mu(x, y, 0)$. In both cases, we can show that the attenuation expression at point $M(x, y, z)$ can be deduced from that calculated at point $(x, y, 0)$ by

$$(11) \quad \begin{aligned} \exp\left(-\int_{M_{det}}^{M(x, y, z)} \mu(l) dl\right) &= \exp\left[-\mu_{cont} \cdot \|C_{ji}^{ext} - C_{ji}^{int}\| - \mu_{obj} \cdot \|G_{ji} - C_{ji}^{int}\| \cdot \sqrt{1 + \frac{z^2}{\|M_{det} M\|^2}}\right] \\ &= \exp\left(-\int_{M_{det}}^{M(x, y, 0)} \mu(l) dl\right) \cdot \exp\left[-\mu_{obj} \cdot \|G_{ji} - C_{ji}^{int}\| \cdot \left(\sqrt{1 + \frac{z^2}{\|M_{det} M\|^2}} - 1\right)\right] \end{aligned}$$

We have performed a certain number of simulations using different attenuation values, in order to compare an LSF calculated with the relation (8) and an LSF where the attenuation is calculated according to (9). The results obtained show that hypothesis $[h_0]$ is a good approximation in the case of a homogeneous density. Taking into account the distance variation along the line within a homogeneous material does not provide significant variations on all the calculation space and whatever the attenuation value.

$$(12) \quad res_{rel}(\mu) = 100 \frac{\|LSF(\mu) - LSF_0(\mu)\|}{\|LSF(\mu)\|}$$

for nine values of μ included in $[0, 0.3]$. The residue is not nil when $\mu=0$ because the model form (8) is computed using the numerical integral method, therefore of finite accuracy, whereas (9) is made using an exact calculation. The calculations were made over 64^2 points covering the theoretical detection space.

Table 1: Relative differences between two LSF calculation methods

$\mu \text{ (cm}^{-1}\text{)}$	0	0.0001	0.001	0.01	0.03	0.05	0.08	0.10	0.20	0.30
$res_{rel}(\mu) \text{ (%)}$	$6.6e^{-13}$	0.058	0.063	0.066	0.068	0.067	0.067	0.067	0.066	0.065

In order to establish the matricial equation (3), we must linearize equation (7) : this highly depends on the intensity distribution model we are going to choose. Two models, often used in tomography, are the Dirac model and the concave disk. The first assumes, as its name indicates, that all the pixel intensity is concentrated in its center, thus simplifying the modelling of the projection. In the second one, the activity distribution f is uniformly distributed on the disk inscribed in the pixel. The latter, developed in (33) also simplifies the modelling since the shape of the projection is independent from the angle on which it is performed (projecting a cercle results in an unvarying shape whatever the angle). We propose in the following paragraphs, two modellings slightly different from this appearing in (33). We have chosen an homogeneous volume distribution of activity: it occupies all of the voxel volume. The two modellings are different in their way of approaching the surfacic integral calculation. The first method which we shall develop now performs a calculation by barycentric approximation.

We set $G_{kl}(x_{kl}^G, y_{kl}^G, 0)$ the barycentre of V_{kl} . The activity distribution is assumed to be constantly distributed on V_{kl} , so that we note

$$(13) \quad \forall (x, y, z) \in V_{kl}, \quad f(x, y, z) = f_k$$

the equality (7) can be re-formulated as

$$(14) \quad m_l = \sum_{k \in \Gamma_l} f_k \iint_{S_{kl}} \left\{ \int_{[-h, h]} PSF(x, y, z) dz \right\} dx dy = \sum_{k \in \Gamma_l} f_k \iint_{S_{kl}} LSF(x, y) dx dy$$

Moreover, in a second hypothesis we evaluate the integral calculation using the barycentre value, and expressing the LSF with equation (9):

$$(15) \quad m_l \approx \sum_{k \in \Gamma_l} f_k \cdot \iint_{S_{kl}} LSF(G_{kl}) dx dy = \sum_{k \in \Gamma_l} f_k \cdot \left[\exp \left(- \int_{M_{det}}^{G_{kl}} \mu(l) dl \right) \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{det}^p(x_{kl}^g, y_{kl}^g, z) \cdot dz \right] S_{kl}$$

This calculation requires the determination of quantities G_{kl} , S_{kl} and of course the knowledge of $R_{\det}^p(M)$. We will detail the determination of the quantity $R_{\det}^p(M)$ later. The quantities G_{kl} , and S_{kl} are accessed using the n_{kl} vertex of S_{kl} , $3 \leq n_{kl} \leq 6$. The coordinates of the vertices are obtained through the intersection of S_k with the object outline $\partial\Omega$ and with the kernel K_l :

$$(16) \quad S_{kl} = S_k \cap \left\{ (x, y) \in [-R_{obj}, R_{obj}]^2 / x^2 + y^2 \leq R_{obj}^2 \right\} \cap K_l$$

The polygon S_{kl} is then decomposed in $(n_{kl} - 2)$ triangles, which allows a rapid extraction of both the total surface and barycentric coordinates. In (34), the author proposes a use of pixel-surface calculations within the framework of parallel kernel detections.

By identifying (15) and (3), we deduce the coefficient H_{lk} , influence of the voxel k on the measurement l :

$$(17) \quad H_{lk} = S_{kl} \cdot \exp \left(- \int_{M_{\det}}^{G_{kl}} \mu(l) dl \right) \cdot \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{\det}^p(x_{kl}^g, y_{kl}^g, z) \cdot dz$$

We can however remain doubtful to the domain of validity of these hypotheses and on the accuracy of the calculation of H_{lk} depending on the voxels size. We may cope with that problem by proposing an increase of calculation points on the basis S_{kl} , thanks to a over-sampled second grid of finite elements, including $N' = \tau^2 N$ voxels, where t is an integer greater or equal to 2.

The notations remain the same, and indexes corresponding to the over-sampled grid are noted with « ' ». We define $\Lambda_k(\tau) = \left\{ k' \in [1, \tau^2 N] / V_k = \bigcup_{k'} V_{k'} \right\}$ the set of voxel indexes making by their union the voxel k , and $\Lambda_{kl}(\tau) = \left\{ k' \in [1, \tau^2 N] / (x_{k'}^g, y_{k'}^g) \in S_{kl} \right\}$ the points sub-set of $\Lambda_k(\tau)$ which centers $(x_{k'}^g, y_{k'}^g)$ are contained in S_{kl} .

If we provide ourselves with a sufficiently fine grid-subdivision, all the voxels thus retained form a good approximation of the initial volume calculation V_{kl} (there is also convergence in surfacic terms), in other words :

$$(18) \quad \lim_{\tau \rightarrow +\infty} \bigcup_{k' \in \Lambda_{kl}(\tau)} V_{k'} = V_{kl} \Leftrightarrow \lim_{\tau \rightarrow +\infty} \bigcup_{k' \in \Lambda_{kl}(\tau)} S_{k'} = S_{kl}$$

Thus, if we set t sufficiently great so that all the voxels belonging to $\Lambda_{kl}(\tau)$ make up a good approximation of surface S_{kl} , equality (14) is re-written :

$$(19) \quad m_l = \sum_{k \in \Gamma_l} f_k \cdot \sum_{k' \in \Lambda_{kl}(\tau)} \iint_{S_{k'}} LSF(x, y) dx dy \approx \sum_{k \in \Gamma_l} f_k \cdot \sum_{k' \in \Lambda_{kl}(\tau)} LSF(x_{k'}^g, y_{k'}^g) \cdot S_{k'}$$

Finally, the influence of the voxel k on the measurement l using the over-sampled grid is

$$(20) \quad H_{lk} = \sum_{k' \in \Lambda_{kl}(\tau)} S_{k'} \cdot \exp \left(- \int_{M_{\det}}^{G_{kl}} \mu(l) dl \right) \cdot \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{\det}^p(x_{k'}^g, y_{k'}^g, z) dz$$

In relations (17) and (20), we still have to determine $R_{\text{det}}^p(M)$:

From the relation established in (6), the PSF and $R_{\text{det}}^p(M)$ coincide in absence of attenuation. The spatial response expression of the collimated detector is given from the solid angle seen from point $M(x,y,z)$, so :

$$(21) \quad R_{\text{det}}^p(M) = \frac{1}{4\pi} \iint_{S_M} \frac{d\vec{M} \cdot d\vec{S}}{\|MS\|^2} = \frac{1}{4\pi} \iint_{S_M} \frac{\cos\chi}{(u-x)^2 + (D_{\text{det}} + y)^2 + (w-z)^2} dudw$$

S_M is the domain of surface $l_m \cdot h_m$ belonging to the detection plane observable from point M ,

$d\vec{S}$ is the normal unit vector on the detector plane,

$d\vec{M}$ is the unit vector of the line connecting M to S ,

$\cos\chi$ is the angle formed between $d\vec{S}$ and $d\vec{M}$ (c.f. Figure 2 below)

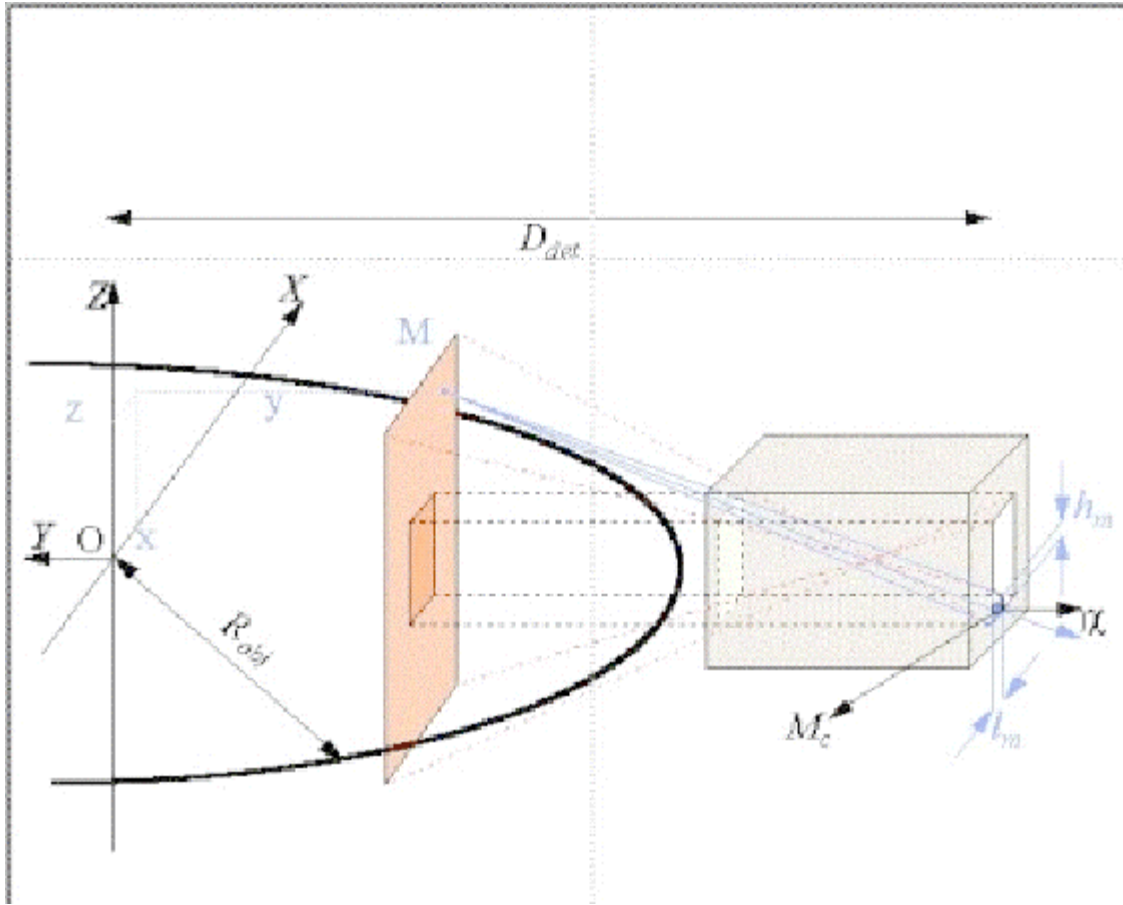


Figure 2 : Solid angle generated by point M

The great distance between the source space Ω and the detection plane, as well as the relatively small angle aperture, allows us to reasonably consider the term $\cos\chi$ as close to the unity, simplifying (21) to

$$(22) \quad R_{\text{det}}^p(M) \approx \frac{1}{4\pi} \iint_{S_M} \frac{dudw}{(u-x)^2 + (D_{\text{det}} + y)^2 + (w-z)^2}$$

A numerical study (conducted using the LSF) shows that the maximum relative error between the response integrating $\cos \chi$ and the spatial response with the simplification remains lower than 0.25%. We can also show that the distance $\|MS\|$ only slightly varies and can be approached by that taken at the central point $M_c(x_c, -D_{\text{det}}, z_c)$ of the rectangle S_M . At this moment, we obtain

$$(23) \quad R_{\text{det}}^p(M) \approx \frac{l_m(M) \cdot h_m(M)}{4\pi \cdot \|MM_c\|^2}$$

This last expression is particular as its integration according to z leads to a literal expression. At distance y from the detector plan, a source is detected up to a height $h(y)$ such as

$$(24) \quad h(y) = \frac{h_{\text{col}}}{L_{\text{col}}} \cdot (D_{\text{det}} + y - \frac{L_{\text{col}}}{2})$$

As the calculation steps are detailed in (35), we only give the final result here

$$(25) \quad \int_{z=-\frac{h}{2}}^{\frac{h}{2}} R_{\text{det}}^p(x_{k'}^g, y_{k'}^g, z) \cdot dz = \frac{l_m(x_{k'}^g, y_{k'}^g)}{2\pi} \left\{ \frac{h_{\text{col}}}{c_1} \arctan\left(\frac{h_{\text{col}}}{2c_1}\right) + \frac{h_{\text{col}}}{ac_1} \left[\arctan\left(\frac{ah(y_{k'}^g) + b}{c_1}\right) \right]_{a\frac{h_{\text{col}}}{2}+b}^{a\frac{h(y_{k'}^g)}{2}+b} \right\} \\ - \frac{l_m(x_{k'}^g, y_{k'}^g) \cdot L_{\text{col}}}{4\pi a^2 (y_{k'}^g - L_{\text{col}})} \left\{ \ln \left[\frac{c_1^2 + (ah(y_{k'}^g) + b)^2}{c_1^2 + \left(a\frac{h_{\text{col}}}{2} + b\right)^2} \right] - \left[\frac{2b + ah_{\text{col}}}{c_1} \arctan\left(\frac{u}{c_1}\right) \right]_{a\frac{h_{\text{col}}}{2}+b}^{a\frac{h(y_{k'}^g)}{2}+b} \right\}$$

with

$$c_1^2 = (x_c - x_{k'}^g)^2 + (y_{k'}^g)^2, \quad a = \frac{2y_{k'}^g - L_{\text{col}}}{2(y_{k'}^g - L_{\text{col}})} \text{ and } b = -\frac{L_{\text{col}} h_{\text{col}}}{4(y_{k'}^g - L_{\text{col}})}$$

EXPERIMENTAL VALIDATIONS OF THE SPATIAL DETECTOR RESPONSE

The linear sources used in the experiment are 0.8-cm diameter ^{152}Eu sources, 73-cm long. These sources are multi-energetic which allows us to study the model at different attenuation levels. In order to study the limits of the model at very low energy we will also use a ^{241}Am point source with a total absorption peak at 59 keV.

Validation of the theoretical point response model

The validation of the mathematical expression obtained in (23) required the use of Americium^(iv) and Europium^(v) point sources in order to cover a large energy domain. The source was successively placed at a minimum distance, $y_{\text{min}} = D_{\text{det}} - R_{\text{obj}}$, in the center $y_{\text{moy}} = D_{\text{det}}$, and at maximum distance

$y_{\max} = D_{\text{det}} + R_{\text{obj}}$. On each of the planes, we used the symmetry of the acquisition geometry to reduce the sampling to the positive quarter : $x^3 0, z^3 0$.

In order to approach the possible phenomena of septas penetration, we opted for a maximum course superior to 10%, and the handlings were done without attenuation and without any shield, thus improving the counting statistic. Model (23) does not take the septas penetration into account, since these experiments were only conducted to validate the geometrical model. They however allowed us to show the existence of the phenomenon and to quantify its importance. A scale factor was needed to readjust the theoretical data to those from the experiment. This factor was chosen so as to realize the minimum mean square error between experimental and theoretical data.

Remark : We will again find in the next paragraph the need to apply a calibration factor when determining the linear response. At this level, we will see that the calibration characterization is of utmost importance if we want to reach the objectives in terms of quantification on the global activity in tomographic reconstruction.

A three dimensional representation of the theoretical response is given in Figure 3 (the two graphs from the left) in the presence of a nil attenuation and a 0.1 cm^{-1} attenuation (corresponding to the attenuation the radionuclide Cs^{137} [662 keV] would be submitted to in a homogeneous sand matrix). The positive quarter has been eliminated for an internal visualization.

Remark : The pseudo-colors intensity scales could not be set identically on the two graphs as there is a factor of 10^4 between the two.

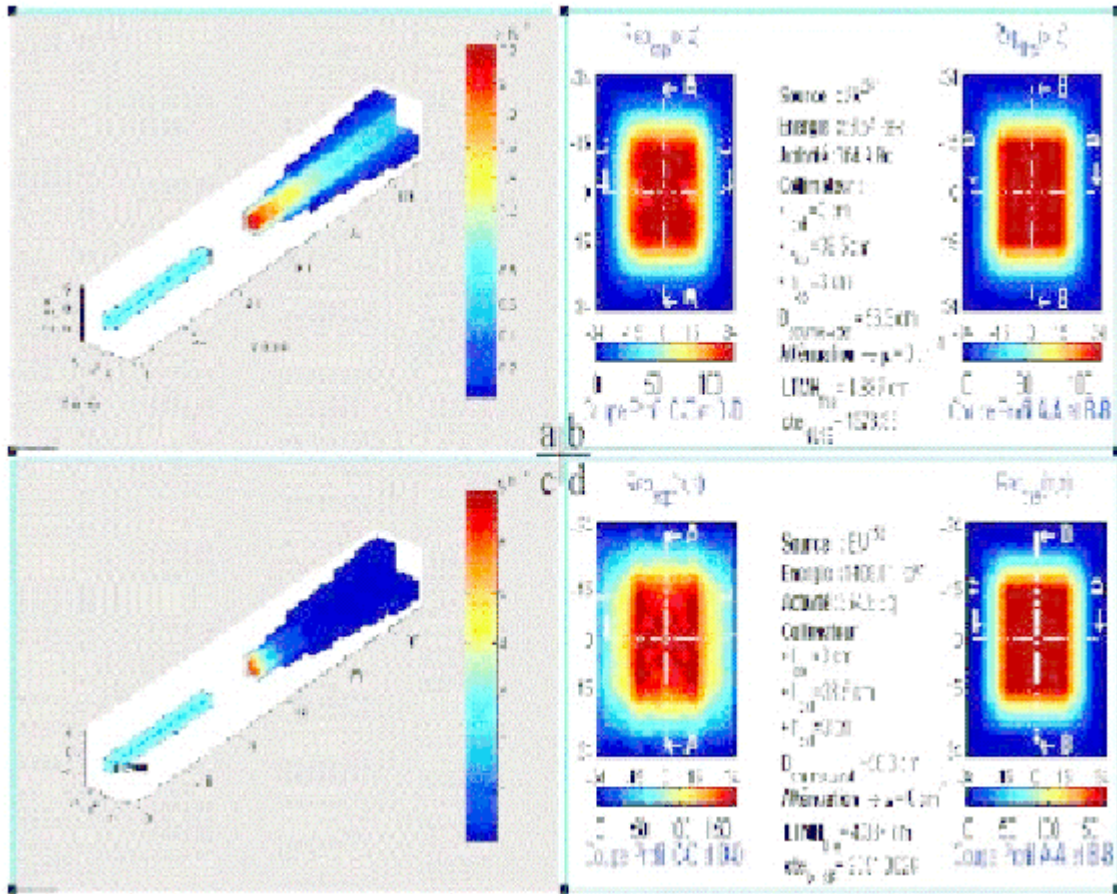


**Figure 3 : a) and c) Point response at $m=0 \text{ cm}^{-1}$ and $m=0.1 \text{ cm}^{-1}$
b) and d) : Experimental responses (left) and theoretical responses (right) on ^{241}Am and ^{152}Eu
point sources at $y_{\min}$**

The PSF behaviour on 3-c allows us to better understand the difficulty in reconstructing matrices with high densities. The manipulations using point sources have allowed us to validate equation (23) on 3 source-to-detector distances, and for 8 different energies, in other words 24 cases. All of them indicate a good prediction of the phenomenon and the quadratic residual errors range from 0 to 28%. The maximum error being reached at the maximum energy peak (source: ^{152}Eu - peak: 1408 keV, Figure 3-d) for a minimum source-detector distance. The quadratic residual error was calculated respectively according to two cross section axes (A-A for Z-axis and C-C for X-axis) on windows 3-b and 3-d. In 3-b, an americium source placed at a minimum distance of 56 centimeters : the prediction is of very good quality, which is normal in view to the fact that the photonic flux by penetration through the collimator lead walls is nearly non-existent at 59 keV. Even though the collimator hole is square, a slight difference is to be underlined between profiles A-A and C-C. A problem of alignment of the source at the time of measurements could be the cause. In 3-d, the comparison is made using an ^{152}Eu source at 1408 keV placed at a distance $y_{\min}$. The response anisotropy is much more obvious. The error made on profile A-A has been estimated at 15% whereas it reaches 28% according to C-C. It also illustrates the penetration phenomenon through lead which is the cause for the blurring beyond the theoretical detection kernel.

In conclusion, we can establish that the proposed model corresponds to a good prediction of the point response of our experimental device. Thus, we should in theory obtain a modelling of nearly the same quality as from the measurements on the linear distributions. This is the framework of the study in the next chapter.

Extension to linear response and determination of a calibration $c(y, E)$

The experimental linear response was treated from a ^{152}Eu linear source placed at different distances from the detector in the detection kernel. All the handlings mainly reach two goals.

1. A first aim is to verify that the passage to the integration following z , i.e. equation (25), is correct. In other words, that we have correspondence between the experimental linear response and the theoretical linear response,
2. the essential aim is to characterize calibration through numerical interpolation which must be applied in order to obtain a quantitative coherence between modelling and measurements.

We do not mention it here as a major objective, but these handlings indirectly enabled us to analyze the importance of the correction of the penetration effect and of its input in the LSF modelling. A modelling of penetration effect has been elaborated according to the same principle as the establishment of the spatial response from the solid angle. The results are visible in terms of linear response prediction quality. In table 2, the residual norm was calculated over all the experimental points, function of the emission energy, between the experimental LSF and the theoretical LSF (calibration included). The first line corresponds to the case where the theoretical LSF was calculated according to the purely geometrical characteristics (cf. equation (25)). The second line included the septas penetration correction.

Table 2: Influence of the septal penetration factor on the prediction quality of the LSF.

LSF _{exp} versus :	121 keV	244 keV	344 keV	779 keV	964 keV	1112 keV	1408 keV
LSF ₀	68.52	80.48	81.28	121.35	139.53	148.45	178.70
LSF ₀ +LSF _{sep} ta	66.84	72.06	63.64	66.57	73.36	79.02	102.48

The improvement of the theoretical prediction is very clear on the results of Table 2. The modelling, of which the bases are presented in (35), only finds its value through the experiments conducted on the linear sources. The septas penetration remains negligible compared to the main photonic flux and experiments concerning it would be time consuming in view of the limited flux : no experiment on mock-up has been performed in order to study this model on quantification and localization. Our studies are not carried out in this direction now, however a series of experiments on linear sources with plugged collimation should enable a better confrontation of the model with reality, and possibly its improvement.

The calibration is determined as follow:

The calibration $c(y_i, E_j)$ is the real positive coefficient achieving the minimum error between the experimental profile obtained from the spectrum of the j 'th isotope for a source placed at a distance y_i .

The theoretical points are calculated from expression (25),

$$(26) \quad c(y_i, E_j) = \min_{c \geq 0} \left(\sum_{x_{k,i}} \left(c \cdot LSF_{the}(x_{k,i}, y_i, E_j) - LSF_{exp}(x_{k,i}, y_i, E_j) \right)^2 \right)$$

LSF_{the} and LSF_{exp} designate respectively the theoretical linear response and its experimental equivalent

Given $r(y_i, E_j)$ the residue for calibration $c(y_i, E_j)$ defined by :

$$(27) \quad r(y_i, E_j) = \sum_{x_{k,i}} \left(c(y_i, E_j) \cdot LSF_{the}(x_{k,i}, y_i, E_j) - LSF_{exp}(x_{k,i}, y_i, E_j) \right)^2$$

Let's work at fixed energy E_j : the expression of $c(y_i, E_j)$ is approximated by a polynomial interpolation in y of degree 3 weighted by $r(y_i, E_j)$. If we note : $P_j(y) = a_j y^3 + b_j y^2 + c_j y^1 + d_j$ this polynome, the vector of variables (a_j, b_j, c_j, d_j) verifies

$$(28) \quad (a_j, b_j, c_j, d_j) = \min_{a_j, b_j, c_j, d_j} \left(\sum_{y_i} \frac{1}{r(y_i, E_j)} (a_j y^3 + b_j y^2 + c_j y^1 + d_j - c(y_i, E_j))^2 \right)$$

The vectors $(a_j, b_j, c_j, d_j)_{j=1..7}$ are then interpolated in energy, element per element, and we note

1. $A(E) = \sum_{i=0..3} A_i E^i$ the polynomial interpolation of degree 3 of the couples of points $(a_j, E_j)_{j=1..7}$,
2. $B(E) = \sum_{i=0..3} B_i E^i$ the polynomial interpolation of degree 3 of the couples of points $(b_j, E_j)_{j=1..7}$,
3. $C(E) = \sum_{i=0..3} C_i E^i$ the polynomial interpolation of degree 3 of the couples of points $(c_j, E_j)_{j=1..7}$,
4. $D(E) = \sum_{i=0..3} D_i E^i$ the polynomial interpolation of degree 3 of the couples of points $(d_j, E_j)_{j=1..7}$,

From this, we determine a polynomial function of E and y of our calibration by the formula :

$$(29) \quad c(y, E) = A(E)y^3 + B(E)y^2 + C(E)y^1 + D(E)$$

EXPERIMENTAL VALIDATION ON 200-L MOCK-UP

A comparison of the reconstructed activities compared to the real one allows us to demonstrate the efficiency of the emission tomography in the presence of a small number of radioactive heterogeneities versus a global measurement method such as MARCO-CARACO.

In order to do this, a package configurable in matrix and in radial inhomogeneous source was equipped with a linear source positioning system. The latter and the sources are immersed by the chosen matrix. The reference sources of ^{152}Eu have an average activity of 10 Mbq on the 03/04/96 distributed over 73-cm long. The matrices used correspond to polyethylen-balls for an average density equal to 0.54 ± 0.04 and fine grain sand with an average density equal to 1.46 ± 0.11 . The absorption of the γ radiation in the matrix is characterized by the undimensionnal product of the linear attenuation μ at the energy studied by the internal radius R_0 of the package, (i.e. $R_0 = R_{obj} = 28.5$ -

cm). Seven peaks are studied distributed in the energy window [122, 1408 keV] in other words a field μR_0 included in the interval [1, 7].

At table 3, we list the positions and the activities corresponding to the different configurations used. The activities are given with a relative uncertainty of 5%.

Table 3 : Total and linear activities for 4 different configurations

N° source	Polar Positions	Lin. Act. MBq/cm (3/4/96)	1 source MBq (3/4/96)	3 sources MBq (3/4/96)	5 sources MBq (3/4/96)	7 sources MBq (3/4/96)
102	D0	0.138	10.1	10.1	10.1	10.1
103	D1-A7	0.134		9.8	9.8	9.8
104	D3-A5	0.167		12.2	12.2	12.2
105	D4-A4	0.167			12.2	12.2
106	D5-A3	0.162				11.8
107	D6-A2	0.160			11.7	11.7
108	D7-A1	0.148				10.8
Total MBq :			10.1	32.1	56.0	78.6
Total linear MBq/cm :			0.138	0.440	0.767	1.077

A global measurement method : MARCO-CARCO

The SCCD/LEC has developed a measurement and correction method called MARCO-CARACO (36) which calculates a transfer function which can be parameterized for all cylindrical-types packages.

The measurement phase (MARCO) consists in integrating the flux coming from all of the cylindrical package rotating during the time of measurement. It is placed at a distance at least equal to five times the radius in order to assimilate the package to a point source and thus averaging the flux in the presence of inhomogeneities. The detection device is first calibrated using point sources placed one meter from the detector.

The quantification method (CARACO) consists in modelling the matrix by a flat shield with the same characteristics in terms of mass attenuation and volume mass as the initial matrix assumed to be physically homogeneous. Behind this equivalent shield, a linear source is placed with activity to be quantified corresponding to the activity contained in the package and assumed to be homogeneously distributed in the matrix. This method corresponds to the «Rockwell approximation» (37), and is only valid if the package is sufficiently far from the detector.

When the package fabrication process corresponds to a homogeneous waste mixture with an embedding matrix and if this process can be reproduced, then this measurement method is specially adapted to the measurement of the final package.

In the case of unembedded packages, a specific study consisted in estimating the bias generated by this method in the presence of heterogeneities (38). It seems that the most penalizing radioactive heterogeneities and more particularly their radial distribution which can generate biases of several orders of magnitude on the calculated activity if the package is absorbent .

A Ray-Driven-type segmented method

The "ray-driven" modelling consists in calculating the projection matrix H , where the determination of the elements is limited to the central ray detection. Compared to the volumic method described above, the detection nucleus K_l reduces to the central linear beam (passing through M_{det}) and a assimilable Dirac model is chosen for the intensity distribution. The calculation method used is described in (32).

The method consists in a first step, in calculating the intersection of the central ray with elements of the N pixels-grid of the image $\mu(x,y)$ following the lines or the columns (depending on the projection angle θ) in order to favor the axis on which there are the most intersections. The second step performs integration calculations for each intersection point by interpolating between the neighboring pixels if the ray does not cross the pixel in its center and by taking into account the distance from the detector used (law in $1/y^2$). A specific treatment is used for the pixels located on the edges of the object. Numerous elements of the matrix remain nil as few pixels contribute in a measurement (H is sparse and consequently difficult to inverse).

The «ray-driven» type algorithm assumes that the measurement is made along a straight line without thickness. In fact, the measurement integrated a certain width (beam) which explains the importance in adjusting the pixel size to be reconstructed which must be greater or equal to the sampling of the projections. This type of modelling does not take into account the beam divergence induced by the solid angle of the collimator. An iterative algebraic method called ART (Algebraic Reconstruction Technique) is used to find the solution. We estimate successive values of $\mathbf{f}$: when an image has been estimated, it is successively retro-projected on the hyperplanes formed by the lines of the matricial system (i.e. equation 29). We use the difference between the measurement vector thus estimated with the real measurements to update the image estimate :

$$(29) \quad f_k^{p+1} = f_k^p + \alpha \frac{H_{lk} \left(m_l - \sum_i H_{li} f_i^p \right)}{\sum_i H_{li}^2}$$

The first use of ART was made in (39). The variations and the properties are described in detail in (40).

The algorithm used for the volumic model is an EM-type (Expectation-Maximization) algorithm. By using the Poisson-distribution, the EM algorithm produces a sequence of images which converge to the maximum likelihood estimate (MLE) of the mission densities. Indeed, a plausible solution to our reconstruction problem is to find the maximum following $\mathbf{f}$ of the conditional likelihood probability $\Pr(\mathbf{m}|\mathbf{f})$. The estimator of the maximum likelihood $\mathbf{f}_{ML}$ therefore requires the solving of the following optimization problem

$$(30) \quad \mathbf{f}_{ML} = \arg \max_{\mathbf{f} \geq 0} \Pr(\mathbf{m}|\mathbf{f})$$

The first reference to the use of the expectation maximization in tomography was made in (41). Later, (42) propose at first the use of the maximization expectation algorithm developed in (43), which is a specific iterative method allowing to reach the expectation maximization of a vector of random variables conditionally to another. If we consider a poissonian likelihood, we will have to maximize the following functionnal:

$$(31) \quad \mathbf{f}_{ML} = \arg \max_{\mathbf{f} \geq 0} \left\{ \sum_l \left(m_l \log \left(\sum_k H_{l,k} f_k \right) - \sum_k H_{l,k} f_k \right) \right\}$$

The iterative diagram of the (p+1)th estimate of the EM algorithm is given by

$$(32) \quad \mathbf{f}^0 \geq 0, \forall k \quad f_k^{p+1} = \frac{1}{\sum_l H_{l,k}} \sum_l m_l \frac{H_{l,k} f_k^p}{\sum_m H_{l,m} f_m^p}$$

Though the quality of the MLE image produced by this algorithm is very encouraging, it has not been found suitable for routine practical applications owing to a slow convergence rate and undesirable tendency to amplify artefacts with the iterations.

We suggest a Bayesian formulation for incorporating priors spatial constraints into the reconstruction (44). These methods allow for the introduction of prior information into the reconstruction problem thereby extending ML approaches via a Maximum *A Posteriori* (MAP) approaches. The distribution of $\mathbf{f}$ conditioned by $\mathbf{m}$ by the Baye's formula is:

$$(33) \quad \Pr(\mathbf{f}|\mathbf{m}) = \frac{\Pr(\mathbf{m}|\mathbf{f}) \Pr(\mathbf{f})}{\Pr(\mathbf{m})}$$

We will use a Gibbs distribution associated with a Markovian model (i.e. the Hammersly-Clifford theorem in (45)), because they allow us to model a large variety of images. Their general form is

$$(34) \quad \Pr(\mathbf{f}) = \frac{1}{Z} \exp(-\beta E_p(\mathbf{f}))$$

where β and Z are constants and $E_p(\mathbf{f})$ represents the potential energy function of the system (46). Its form depends on the Markov random field (MRF) that we choose. In our case we use a second order MRF, constituted by height neighbours, and we limit the dependency at two adjacents pixels. The MAP energy is then expressed by the new functionnal

$$(35) \quad \mathbf{f}_{MAP} = \arg \max_{\mathbf{f} \geq 0} \left\{ \sum_l \left(m_l \log \left(\sum_k H_{l,k} f_k \right) - \sum_k H_{l,k} f_k \right) - \beta E_p(\mathbf{f}) \right\}$$

with

$$(36) \quad E_p(\mathbf{f}) = \sum_{i,j} \varphi_c(f_{i,j} - f_{i,j-1}) + \sum_{i,j} \varphi_c(f_{i,j} - f_{i-1,j}) + \sum_{i,j} \varphi_c\left(\frac{f_{i,j} - f_{i-1,j-1}}{\sqrt{2}}\right) + \sum_{i,j} \varphi_c\left(\frac{f_{i,j} - f_{i+1,j-1}}{\sqrt{2}}\right)$$

we call j_c the potential function associated with $E_p(\mathbf{f})$.

We now differentiate the expression (35) and nullify it in order to solve the normal equations of the system. We uses One-Step-Late (OSL) concept developed in (47) for solving these non-linear equations. By using the last iterate $\mathbf{f}^p$ in the a priori term (instead of the current estimate $\mathbf{f}^{p+1}$), the normal equations are linearised, thus the iterative formula of the MAP-EM-OSL is given by

$$(37) \quad \forall k, \quad f_k^{p+1} = \frac{f_k^p}{\sum_l H_{l,k} + \beta \left. \frac{\partial E_p(\mathbf{f})}{\partial f_k} \right|_{f_k^p}} \sum_l m_l \frac{H_{l,k}}{\sum_m H_{l,m} f_m^p}$$

So at each iteration the MAP-EM-OSL increases the MAP (log-) probability; nevertheless one must be careful with the choice of the potential function since it can render the whole MAP-functional non-convex and thus converge to only local optimum. We choose Green's potential ϕ_{GR} because of his convexity and other numerical characteristics (47-48),

$$(38) \quad \phi_{GR}(x) = 2 \log[\cosh(x)]$$

Results and analyses

We present on the Y-axis the activity ratio between the global reconstructed activity and the known real activity, the whole function of μR_0 (c.f. Figure 4). An equivalent graduation of μR_0 is the mean free path value, noted λ . For exemple, the mean free path values are 9.5 and 4 cm for respectively 3 and 7- μR_0 values.

In the first case, a path of 6λ equals the 200-l drum diameter. In the second case, a path of 6λ is insufficient to measure the activity located in the matrix center.

For the less homogeneous configurations (1 source at the center and 3 sources), ECT provides excellent results until $\mu R_0=4$. MEPHISTO shows a better behavior when reaching this limit over the central ray approximation (ray-driven). Over this limit, insufficient photon counting combined with the unprecision on the lineic attenuation in the photoelectric attenuation domain deteriorate the precision on quantification and localisation. The global method renders ratios systematiquement under 73 %, even for low μR_0 values.

For the 5 and 7-sources configurations, emission tomography and more particularly MEPHISTO enables an optimal quantification until the upper limit of our experiments, which is $\mu R_0=7$.

In the most homogeneous configuration, i.e 7-sources, the global method MARCO-CARACO is applied on a mock-up that verify the assumption of homogeneous radioactive distribution in a precision range of 15%. Thus, if the efficiency of global measurement is experimentally proved in this configuration, it is, according to the 1,3 and 5 source-configurations and for a large range of μR_0 values, inaccurate and inadequate beyond the homogeneity hypothesis. Nevertheless, segmented methods, MEPHISTO as well as the ray-driven method, provide a good estimation of global activity on the 7 sources configuration. A slightly deterioration is yet discernible at the highest attenuation : the main reason is the primordial influence of the object-peripheral source . When passing at the nearest source-to-detector distance, the large volume of the voxel including the source induces a partial volume effect (48). The magnitude of this effect varies with the non-stationnary and anisotropic spatial resolution. Thus, since the variation of the response is maximum at short distance and the fact that the activity repartition is more likely to resemble a Dirac than a evenly voxel spreading, the model inherent to MEPHISTO generates a bias in this case. In the ray-driven as well, a bi-linear interpolation through coarse grid automatically leads to errors.

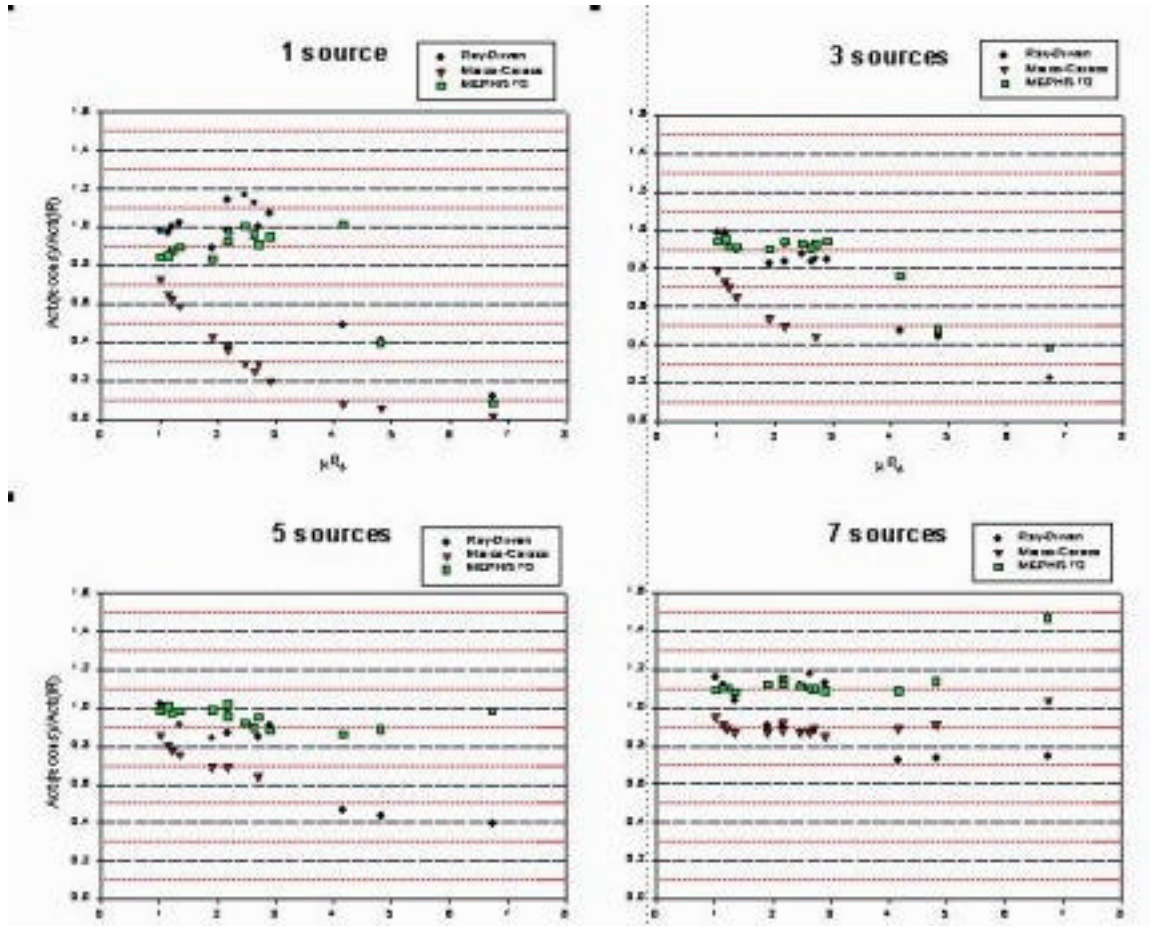


Figure 4 : Ratio results from less to most homogeneous configurations (from left to right, from top to bottom)

CONCLUSION

In this paper, a unified two-dimensional reconstruction method for simultaneous compensation of attenuation, scatter, geometric response and septa penetration is presented. MEPHISTO incorporates a three-dimensional model for the geometric point response in the projector matrix for parallel-hole collimation configuration. It enables to take into account inhomogeneous attenuation by coupling of transmission tomographic maps. The method was tested with mock-up configurations and compared with a central ray approximation and a global measurement method. This method may improve quantification of the radionuclides activity as well as recognition of hot spots in size and shape in the reconstructed image. The modelling of the geometric response is shown to accomplish a smoothing but without any loss of resolution. The nature of the benefit introduced by the consideration of the system response, is clear from Figure 4. It is a typical feature of iterative techniques that beyond a certain number of iterations the quality of images are worsened by artifacts and repercussion is also visible in terms of quantification. The simulations showed that reconstruction with the accurate response correction allows more iterations before the chequerboard effect (inherent to ML reconstructions) begins to appear. This fact is an important point because the gain in numerical stability render the choice of the stopping iteration less critical. Another important point to underline

is the major influence of the mathematical calibration of the HpGe detector. It was typically done with the use of multiple energy calibrations of ^{152}Eu sources placed over a sampling of the detection kernel, without any attenuation factor. Even there, accurate calibration is a difficult task on account of geometrical considerations and random gamma-emission process. It would be worthfull and interesting to analyse the contribution of a proper MCNP (49) simulation of the TEMISEC acquisition system. Monte-Carlo techniques are very powerful and accurate when properly applied. There have been a few very useful publications describing the use of the MCNP tool for the modelling of efficiency and calibration of gamma ray spectra on HpGe detector. This tool could then be used for selecting the optimum collimator size. Since the voxel height depending on h_{col} is responsible of biais when activity distribution is not z-independant, it would be interesting to see if a possible optimal height is foundable.

Furthermore, those experiments underline the difficulty of working with low resolution beam and coarse voxel grid. We have to cope with restricting hypothesis that often do not correspond with reality : Indeed, MEPHISTO's modelling supposes that the activity function is unvaryingly spreaded over the voxel. The veracity of this assumption is naturally more harder to hold when the sampling is getting larger. Therefore, an important work has to be achieved in order to minimize all the possible bias emerging from the differents calculus : the use of multigrid to enforce the estimation of attenuation, the varying detector response, etc. Paradoxaly, a low resolution modelling needs a proportionnaly greater amount of precision, and thus time processing, than for high resolution imagery. The modification of the projection matrix according to the volumic system response using a robust iterative algorithm has indenially improved the quality of reconstruction without completely resolving the problem. It has been noticed that it is possible to get round of the problem of partial volume effect by slightly increasing the number of voxels on the image. Nevertheless, an increasing of the image size does not, at the contrary, resolve this problem, because doing this we automatically degrade the range of matrix H . Yet, we may overcome the difficulty by introducing *a priori* information via the image structure on markovian random fields. This regularisation technique using gibbsian distribution forms the core of our actual development. Further work has to be done in order to assess which regularisation must be chosen in correspondance with the image size and the ill-conditionned matrix H . Many questions are still open, such as the optimal selection (50) of the hyperparameters contained in the given model, such as β , or questions regarding the particular form of the gibbs distribution employed, and the computational expense associated with their implementation.

We also intend to evaluate our models on mock-up including great disparities in terms of physical homogeneities, in order to analyse the influence of taking into account an heterogeneous attenuation map versus a global attenuation estimation.

FOOTNOTES

- (i) MEPHISTO : Multi-Energic PHoton Imagery through Segmented TOMography
- (ii) Direction du Cycle du Combustible / Département Entreposage et Stockage des Déchets / Service de Caractérisation et Contrôle des Déchets
- (iii) Point Spread Function : (point spatial response function of the entire system).
- (iv) emission ray at 59 keV
- (v) main rays inspected : 121 keV, 244 keV, 344 keV, 779 keV, 964 keV, 1112 keV, 1408 keV

REFERENCES

1. Bellini A., Piacentini M, Cafforio C and Rocca F. "Compensation of tissue absorption in emission tomography" . IEEE Trans. Acoustics, Speech and Signal Processing 27(3):213-218, 1979.
2. Glick S, Penney B and King M "Filtering of SPECT reconstructions made using Bellini's attenuation correction method : a comparison of three pre-reconstruction filters and post-reconstruction Wiener filter" IEEE Trans.. Eur. J. Nucl. Med, 26:21,1987
3. Gullberg G and Budinger T. "The use of filtering methods to compensate for constant attenuation in single photon emission computed tomography." Internat.J. of Imag. Syst. and Technol. 28(2):142-157,1989
4. Tretiak O and Metz C. "The exponential Radon transform." SIAM J. Appl. Math.,39(2):341-354,1980
5. Chang L. "A method for attenuation correction in radionuclide computed tomography." IEEE Trans. Nucl. Sci. 25 (1):638-639,1978
6. Gullberg G, Zeng G, Tsui B and Hagius J "An iterative reconstruction algorithm for single photon emission computed tomography with cone-beam geometry." Internat. Syst. and technol.,1:169-186,1989
7. Tsui b, Gullberg G, Edgerton E, Ballard J, Perry J, McCartney W and Berg J. "Correction of nonuniform attenuation in cardiac SPECT imaging." J. Nucl. Med. 30 : 497-507, 1989.
8. Maze A. "Iterative reconstruction methods for nonuniform attenuation distribution in SPECT." J. of Nucl. Med. 34 (7),1993
9. Bourguignon M.H,et al. "Correction of attenuation in SPECT with an attenuation coefficient map: a new method." J. Nucl.Biol. Med 37:26-32,1993
10. Gullberg G, Huesman R, Malko J, Pelc N and Budinger T. "An attenuated projector-backprojector for iterative SPECT reconstruction." Phys Med. Biol. 30: 799-816, 1985
11. Koral N, Clinthorne N and Rogers W. "Improving emission computed tomography quantification by compton-scatter rejection through offset windows." Nucl. Instrum. Methods Phys. Res., A242:610-614,1986
12. Frey E and Tsui B. "A practical method for incorporating scatter in a projector-backprojector for accurate scatter compensation in SPECT." IEEE Trans. Nucl. Sci. 40:1107-1116,1993
13. Halama J, Henkin R, Friend L "Gamma camera radionuclide images : improved contrast with energy-weighted acquisition." Radiology,169:533-538,1988.
14. Ljungberg M and Strand S.E . Scatter and attenuation correction in SPECT using density maps and Monte-Carlo simulated scatter functions. J. Nucl. Med.,31:1560-1567,1990.
15. Buvat I, Rodrigez-Villafuerte M, Todd-Potropek, Benali H and Di Paola R. "Comparative assessment of nine scatter correction methods based on spectral analysis using Monte-Carlo simulations." IEEE Trans. Nucl. Med.,36:1476-1488,1995
16. Jaszczak R, Greer L, Floyd C, Harris C and Coleman R. "Improved SPECT quantification using compensation for scattered photons." J. Nucl. Med., 25:893-900,1984.
17. Lewitt R.M, Edholm P.R and Xia W. "Fourier Method for correction of depth dependent collimator blurring." SPIE Med. Imag II : Image Processing, 1092:232-243,1989.
18. Ogawa K, Paek S, Nakajima M, Yuta S, Kubo A and Hashimoto S. "Correction of collimator aperture using a shift-variant deconvolution filter in gamma camera emission CT." SPIE Med. Imaging II, 914:699-706,1988
19. Formiconi A, Pupi A, Passeri A. "Compensation of spatial system response in SPECT with conjugate gradient reconstruction technique." Phys. Med Biol. 34:69-84,1989

20. Green A, Begent R, Bagshawe K. "Maximum likelihood reconstruction incorporating line spread function data." Eur. J. Nucl. Med. 14:226,1988
21. Zeng G.L and Gullberg G.T "Three-dimensional iterative reconstruction algorithms with attenuation and geometric point response correction." IEEE Trans. Nucl. sci. 38(2):693-702,1991
22. Axelsson C and Danielsson P. "Three-dimensional reconstruction from cone-beam data in $O(N^3 \log N)$ time." Phys. Med. Biol.,39(3):477-491, 1994
23. Beck J, Jaszczak R, Coleman E, Starmer C and Nolte L. "Analysis of SPECT including scatter and attenuation using sophisticated Monte-Carlo modelling methods." IEEE Trans. Nucl. sci. 29:506-511,1982
24. Tsui b, Hu H,Gilland D and Gullberg G. "Implementation of simultaneous attenuation and detector response correction in SPECT." IEEE Trans Nucl. Sci. 35 : 778-783, 1988
25. Beekman F.J and Viergever M.A, "Fast SPECT simulation including shape dependent scatter." IEEE Trans. Med. Ima. vol. 14, No 2, June1995
26. Floyd C, Jaszczak R, Greek K and Coleman E. "Inverse Monte-Carlo as a unified reconstruction algorithm for ECT". J. Nucl. Med.27:1577-1585,1986
27. Liang Z, Turkington T.G, Gilland D.R, Jaszczak R and Coleman E. "Simultaneous compensation for attenuation, scatter and detector response for SPECT reconstruction in three dimensions." Phys. Med. Biol. 37(3):587-603,1992
28. Penney B and King M. "A projector backprojector pair which accounts for the 3D depth and distance dependent blurring in SPECT." IEEE Trans Nucl. Sci, 37:681-686,1990.
29. Morel J, Lepy M.C. "Traitement de l'information en spectrometrie gamma ou X". C.E.A Saclay LMRI 1986
30. Singh M. and Horne C. "Use of a Germanium Detector to Optimize Scatter Correction in SPECT." J. Nucl. Med. 28(2):1853-1860,dec 1987
31. Zasadny K.R, Koral K.F, Floyd C. E and Jaszczak R.J. "Measurement of Compton scattering in phantoms by germanium detector." IEEE Trans Nucl. Sci, 37(2)642-646,1990.
32. Joseph P.M. "An improved Algorithm for reprojection rays through pixel images." IEEE Transactions on Medical Imaging, vol MI-1, no 3, November 1982
33. Huesman R.H, Gullberg G.T, Greenberg W.L and Budinger T.F. "Users manual : donner algorithms for reconstruction tomography." Lawrence Berkeley Laboratory, University of California.
34. Guilpart B. "Calcul des profils tomographiques discrets en géométrie parallèle d'une image discrétisée sur une grille cartésienne". Rapport C.E.A-DAM/CE/MES/RC No 002-Sept 1991.
35. Thierry R. Pettier J.L "Modélisation volumique pour la caractérisation de colis de déchets par tomographie d'émission gamma". Note technique (à paraitre) C.E.A DCC/DESD/SCCD. Decembre 1998
36. Chabaliér B. "Non Destructive Determination of Activity", Nuclear Technology Vol. 115 Aug. 1996
37. Huot N, Chabaliér B, Bonifay P."Effets des hétérogénéités radioactives sur les mesures d'activité par la méthode MARCO-CARACO" Note technique C.E.A DCC/DESD/SCCD. Mars 1996
38. Rockwell T. " Reactor Shielding Design Manual", U.S. Atomic Energy Commission Mc Graw Hill Book Company, Inc. 1956
39. Gordon R, Bender R, Herman G.T "Algebraic reconstruction techniques (ART) for three-dimensional electron microscopy and X-ray photography." J. Theoret. Biol. vol 29, pp 471-482, 1970

40. Gordon R, "A tutorial on ART (Algebraic Reconstruction Techniques IEEE Trans. Nucl. Sci. NS-21 78-93 1974
 41. Rockmore A.J, Macovski A. " A Maximum Likelihood approach to Emission Image Reconstruction from Projection." IEEE Trans. Nucl. Sci. vol NS-23, pp 1428-1432,1976.
 42. Shepp L.A Vardi Y " Maximum likelihood Reconstruction for Emission Tomography." IEEE Trans. on Med. Imag.vol MI-1, No 2, pp 113-121, Oct 1982
 43. Dempster A.P, Laird N.M, Rubin D.B "Maximum Likelihood from Incomplete Data via the EM algorithm." J. Royal Statistical Society, B vol 39, pp1-37,1977
 44. Geman S McClure D.E "Statistical methods for tomographic image reconstruction" Proc. 46th session of the ISI, vol 52, 1987
 45. Besag J "On the statistical analysis of dirty pictures" Roy. J. Statis. Soc. 3 259-302,1986
 46. Geman S. and Geman D. "Stochastic Relaxation, Gibbs Distributions, and the Bayesian restoration of images." IEEE Trans. Pattern Anal. Machine Intell. Vol PAMI-6, pp721-741, Nov 1984.
 47. Green P.J "Bayesian reconstruction from emission tomography data using a modified EM algorithm" IEEE Trans. Med. Imaging, Vol MI-9, N°1, pp. 84-93, March 1990
 48. Pretorius P.H et al "Reducing the influence of partial volume effect on SPECT activity quantification with 3D modelling of spatial resolution in iterative reconstruction." Phys. Med. Biol. 43, pp. 407-420,1998.
 49. Briesmeister J.F. "MCNP - A general Monte Carlo N-Particle Transport Code, Version 4A." Report LA-12625-M, Los Alamos National Laboratory
 50. Zhou Zhenyu et al "Approximate Maximum Likelihood Hyperparameter estimation for Gibbs priors." IEEE Trans. Imag. Process. vol6, no 6, June 1997
-